Maths in Hadlow Primary School

This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with the National Curriculum. Through the policy, we aim to link key manipulatives and representations through concreate, pictorial and abstract methods. School wide policies, such as this, ensure consistency of approach, enabling children to progress stage by stage through models and representations they recognise from previous teaching. By providing children with the opportunity to work on different representations of the same mathematical idea allows for mastery of the concepts and fluency.

True mastery aims to develop all children's mathematical understanding at the same pace. As much as possible, children should be accessing the same learning. Differentiation should primarily be through support, scaffolding and deepening, not through task. White Rose Maths is used alongside this document to support children's learning.

As children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However, it is expected that the majority of children in each class will be working at age-appropriate levels as set out in the National Curriculum 2014 and in line with our 'End Point Document'

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. Our number fluency and mastering number documents work in conjunction with the calculation policy to ensure maximum impact for the children and ensure mastery and fluency across all areas of maths.

Consistency in language is essential for pupils to understand the concepts presented in mathematics.

EYFS

In EYFS, the children achieve the Early Learning Goals for Maths, through a wide variety of songs, rhymes, games and activities. In practical activities and through discussion they will begin to use the correct mathematical vocabulary. The lessons are delivered in conjunction with the NCETM's EYFS learning materials.

The NCETM materials uses *Numberblocks*, a pre-school BBC television series aimed at introducing children to early number. The lessons are designed to assist Early Years practitioners to confidently move on from an episode, helping children to bring the numbers and ideas to life in the world around them.

<table>
<tr><th colspan="2">EYFS Statutory Guidance
Mathematics
Developing a strong grounding in number is essential so that all children develop the necessary building blocks to excel mathematically. Children should be able to count confidently, develop a deep understanding of the numbers to 10, the relationships between them and the patterns within those numbers. By providing frequent and varied opportunities to build and apply this understanding - such as using manipulatives, including small pebbles and tens frames for organising counting - children will develop a secure base of knowledge and vocabulary from which mastery of mathematics is built. In addition, it is important that the curriculum includes rich opportunities for children to develop their spatial reasoning skills across all areas of mathematics including shape, space and measures. It is important that children develop positive attitudes and interests in mathematics, look for patterns and relationships, spot connections, 'have a go', talk to adults and peers about what they notice and not be afraid to make mistakes.</th></tr>
<tr><td>Children in Reception will be learning to…
Count objects, actions and sounds.
Subitise.
Link the number symbol (numeral) with its cardinal number value.
Count beyond 10.
Compare numbers.
Understand the 'one more than/one less than' relationship between consecutive numbers.
Explore the composition of numbers to 10.
Automatically recall number bonds for numbers 0–5 and some to 10.</td><td>ELG Number:
Have a deep understanding of number to 10, including the composition of each number.

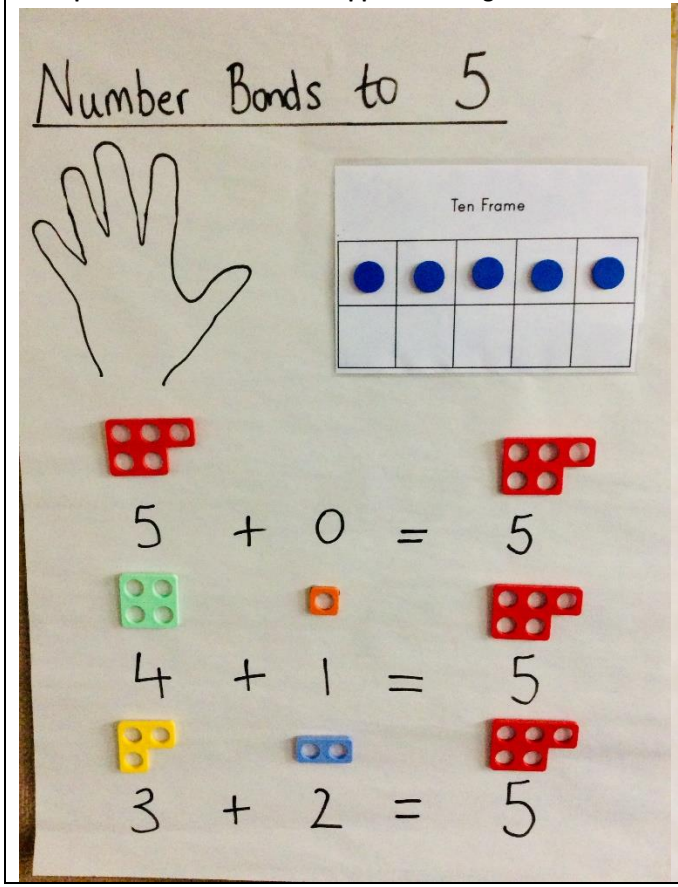
Subitise (recognise numbers without counting) up to 5.

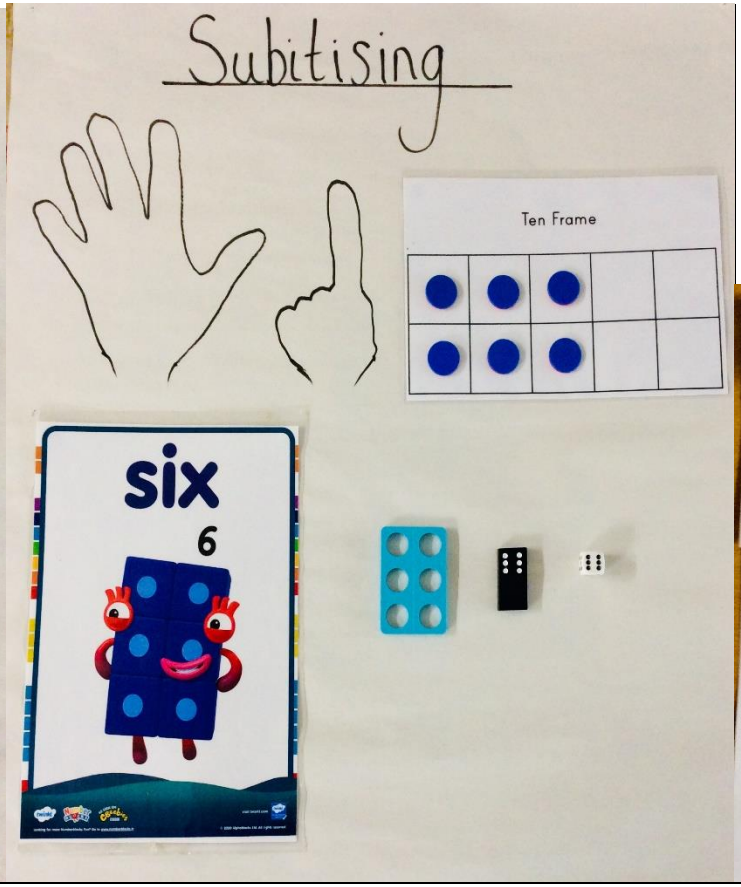
Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.</td></tr>
<tr><td>Children in Reception will be learning to…
Select, rotate and manipulate shapes to develop spatial reasoning skills.
Compose and decompose shapes so that children recognise a shape can have other shapes within it, just as numbers can.
Continue, copy and create repeating patterns.
Compare length, weight and capacity.</td><td>ELG Numerical Patterns:
Verbally count beyond 20, recognising the pattern of the counting system.

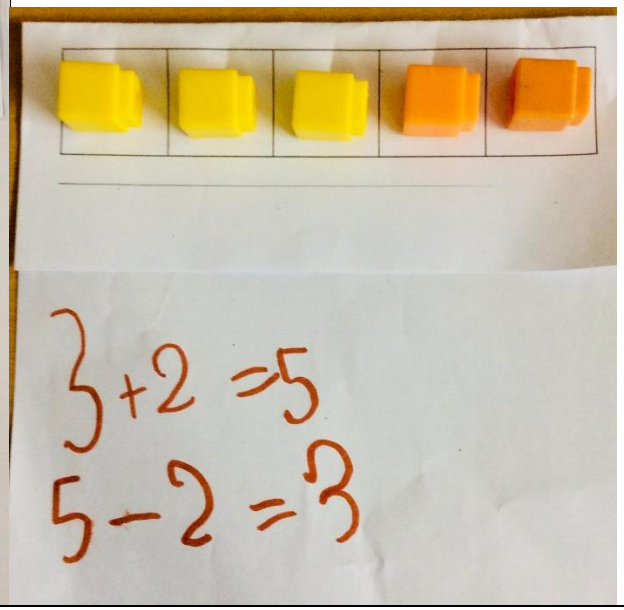
Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than, or the same as the other quantity.

Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed evenly.</td></tr>
</table>

Examples of resources used to support teaching

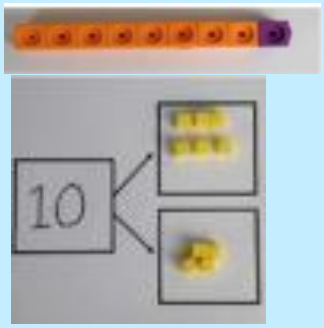
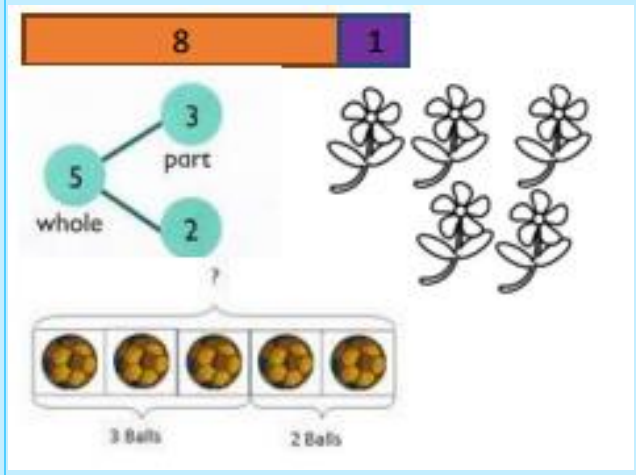
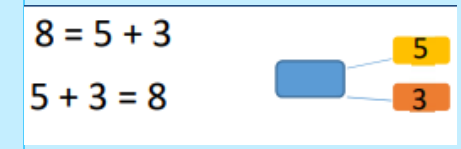
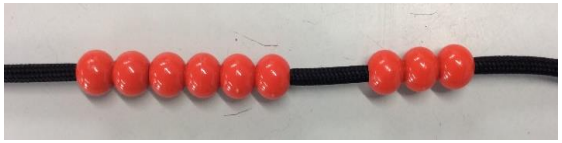
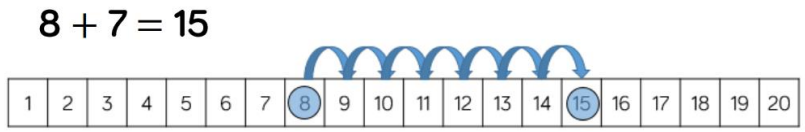


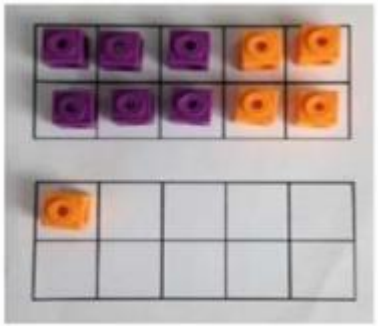
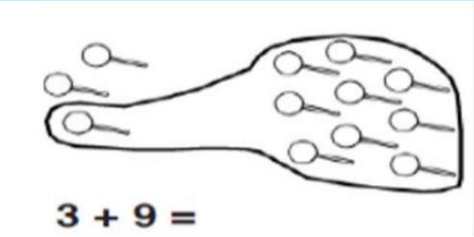
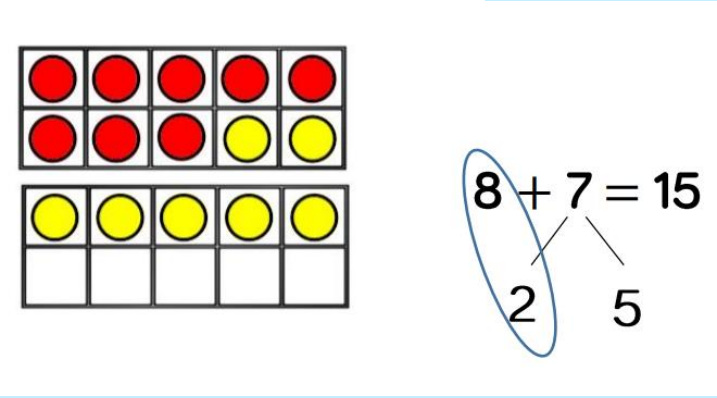
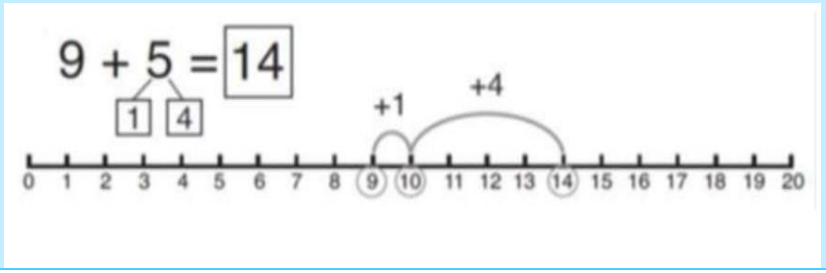


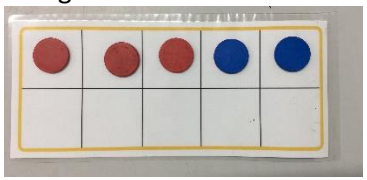
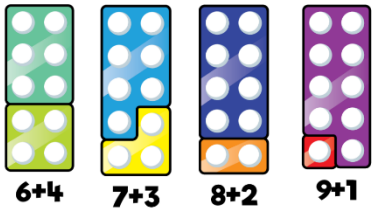
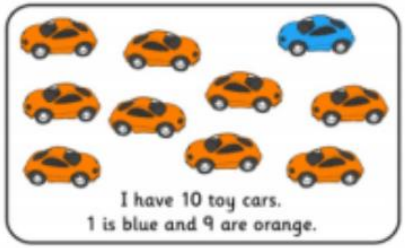
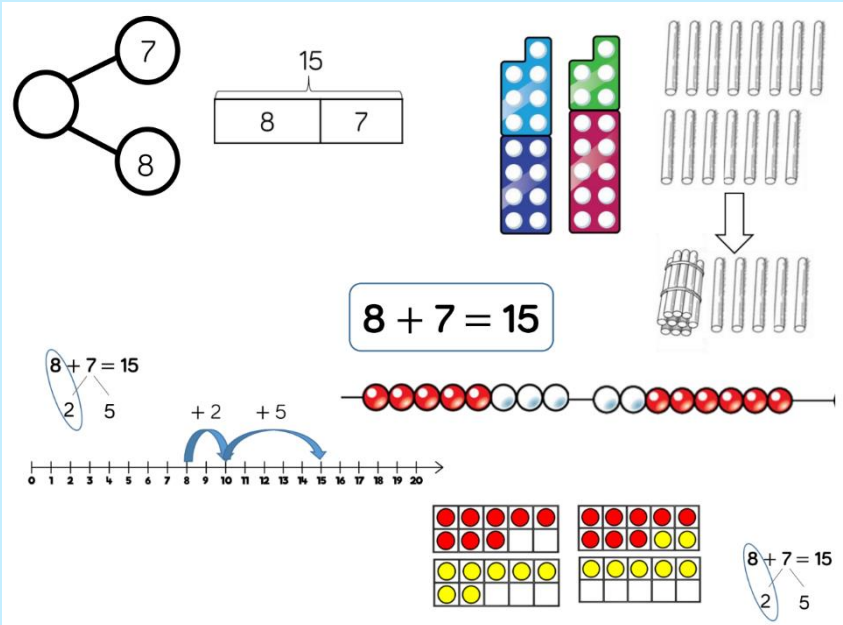


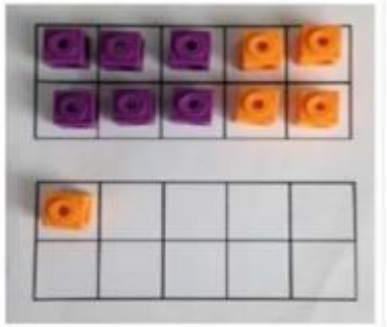
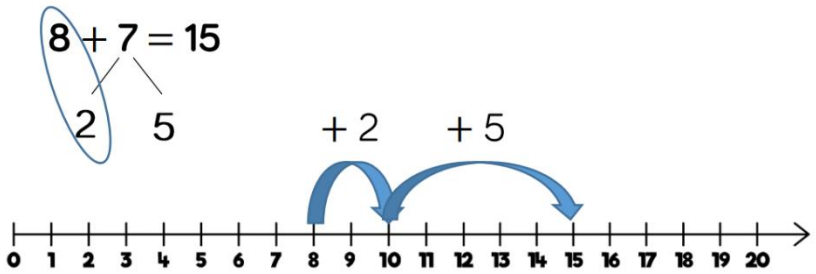
ADDITION

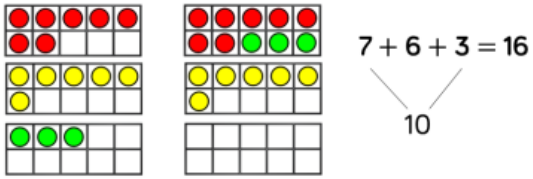
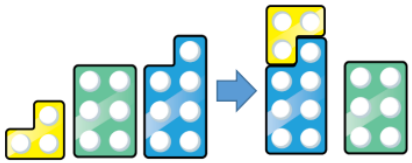
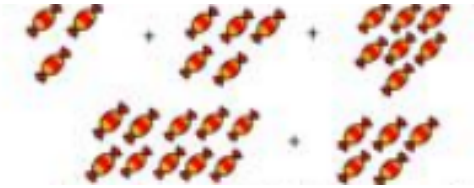
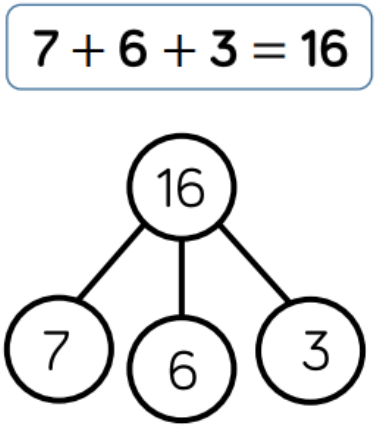
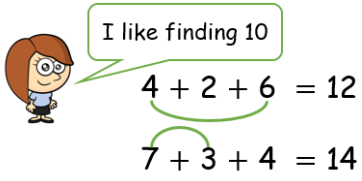
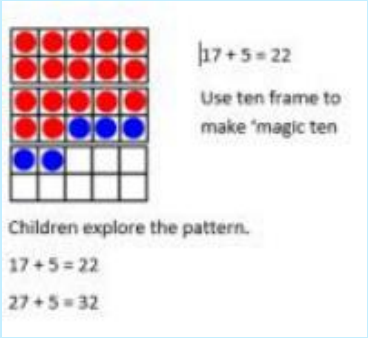
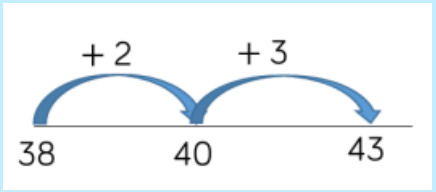
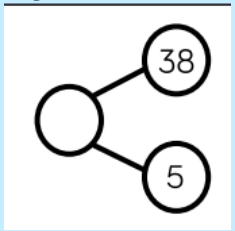
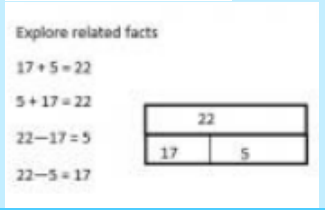
Year 1 Addition

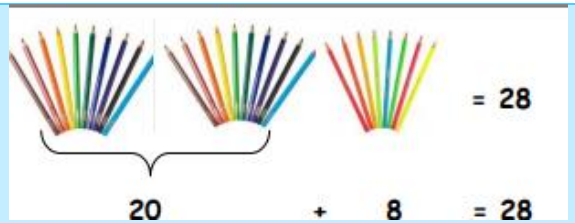
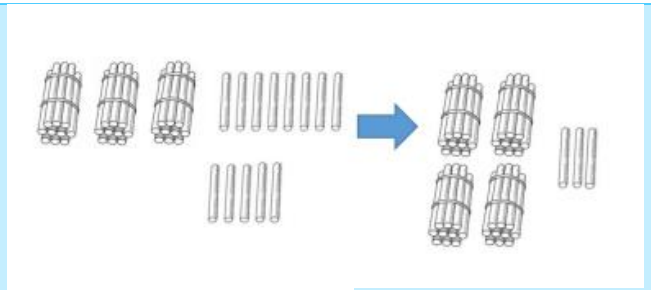
Objective	Concrete	Pictorial	Abstract
Combining two parts to make a whole.	Use cubes, numicon, bead strings, 10's frames and bar models, part, part whole models to recognise when numbers are added they are part of a whole  	Use pictures to add two numbers together as a group. 	Use the part part whole model diagram as shown to help support abstract concept.  Include missing number questions to support varied fluency. **8 =? + 3** **5 + ? = 8**
Starting at the bigger number and counting on	Start with the larger number on the bead sting and then count on to the smaller number 1 by 1 to find the answer 	Start at the larger number on the number line/number track and count on in ones or in one jump to find the answer. 	**5 + 12 = 17** Place the larger number in your head and count on the smaller number to find your answer.

Regrouping to make 10	Start with the bigger number and use the smaller number to make 10. Use ten frames/bead strings.  	Use pictures or a number line. Regroup or partition the smaller number using the part, part whole model to make 10.    	**7 + 4= 11** If I am at seven, how many more do I need to make 10? How many more do I add on now?

Number bonds	Use 10's frames, counters, numicon and bead strings to show number bonds to 5 and 10. 	Use pictures to help support recognising number bonds to 5 and 10.   	**3+7=10** **1+9=10** **10= 8+2** **10=7+3** Include missing number problems **5+ ?=10** **?+ 3=10**
Add one and two digit numbers to 20	Use cubes, numicon, bead strings, 10's frames and bar models, part, part whole models to add two numbers together 	Use pictorial representations to add one or two digits.  	**10+7= 17** **9+7= 16** **12+7=19** Include missing number questions: **8 = ? + 3** **5 + ? = 8** **12+ ?=18** Emphasis should be on the language '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'

Represent and use number bonds and related subtraction facts within 20	Children to use knowledge of number bonds to add two numbers together 	Use number lines to add. 	**7+5=** If I am at seven, how many more do I need to make 10? How many more do I add on now?

Year 2 Addition			
Objective	**Concrete**	**Pictorial**	**Abstract**
Adding 3 one digit numbers	Using tens frames and numicon to help see the patterns to number bonds/near number bonds  	Children to use picture representations to add 3 digits. 	Children to use part whole models to make connections to number bonds to help add.  
Adding a 2 digit number and ones	Children to use 10's frames to make connections of 10's and ones.  	Children can use pictorial representations of number lines/ objects to add   Children can use number bonds to help them bridge 10's	Children to use their knowledge of related facts to help support adding 2 and 1 digit numbers together  

		 |									
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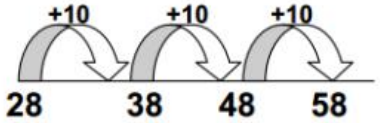
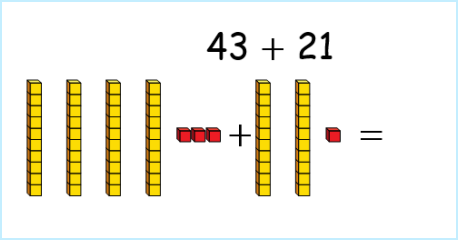
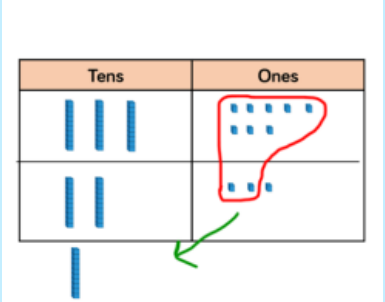
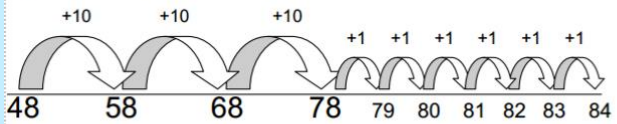
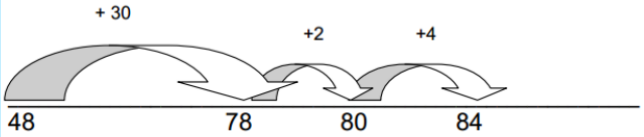
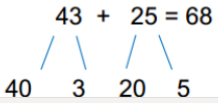
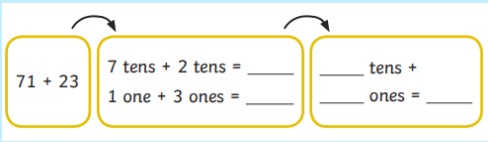
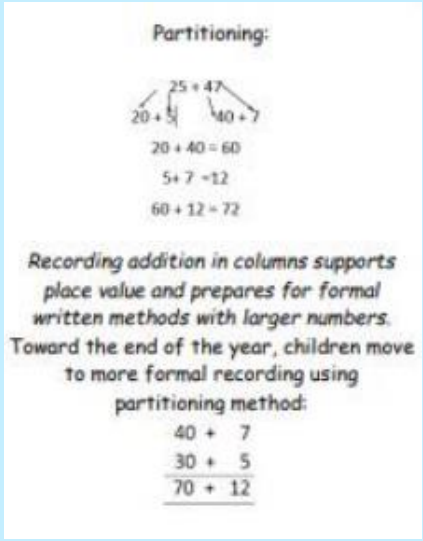
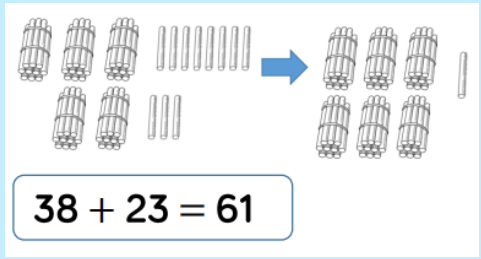
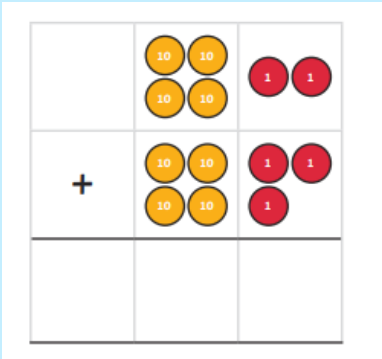
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100		
Adding a 2 digit number and multiples of 10	Numicon, base 10 and counters can be used to visually represent the numbers to the children and they can see the ones do not change.  Explore that the ones digit does not change	Number lines can be used as pictorial representations of adding multiples of 10. **28 + 30 = 58** 	Using place value understanding children can add muliples of 10 and solve missing number problems 27 + 10 = 37 27 + 20 = 47 27 + □ = 57

Adding 2 digit numbers	Counters, bead strings, base 10 and numicon can be used to add 2 digit numbers. Explore not crossing 10 and crossing 10 to work on children's place value knowledge of regrouping.   	Children to use pictures and number lines to explore adding to represent 10's and 1's **48 + 36 = 84** 'Put the biggest number first (48), and then partition the smaller number (36 = 30 + 6) and count on: 48 + 30 + 6.'  Use in conjunction with a **100 square** to show jumps of tens and ones. If children are confident, use more efficient jumps…  Use in conjunction with a **100 square** to show jumps of tens and ones/units. Also use the **partitioning method** to add two two-digit numbers:  	Children to use partitioning to help them break down problems into manageable chunks and to prepare them for formal addition in Year 3. Secure place value knowledge is a must.  


38 + 23 = 61


10
10
10
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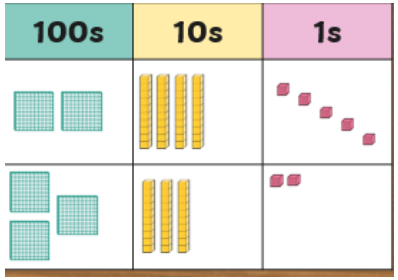
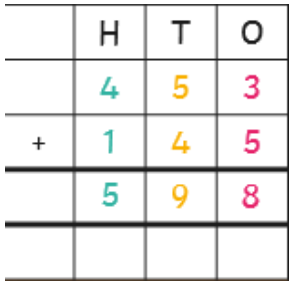
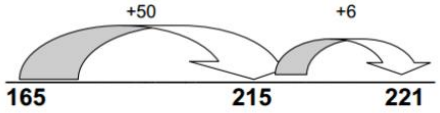
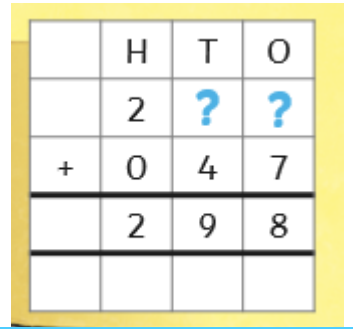
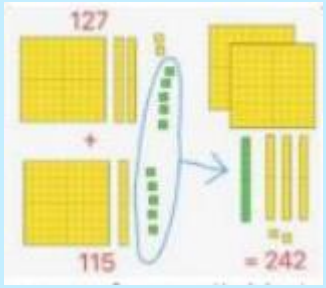
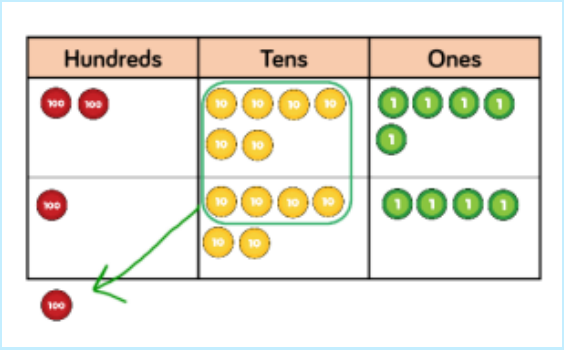
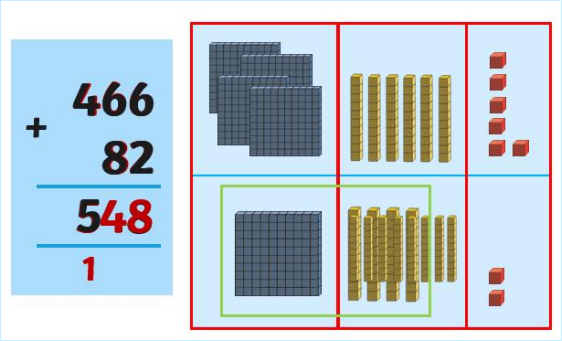
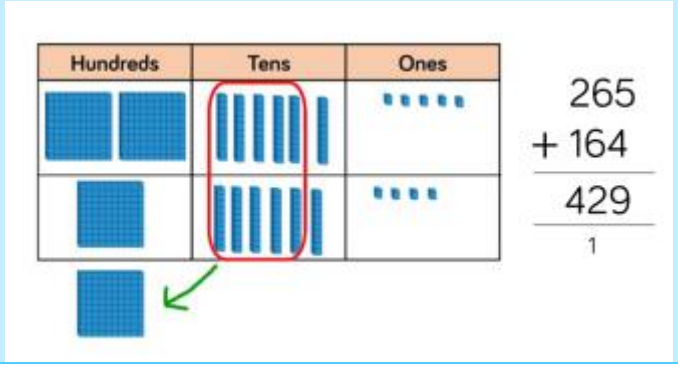
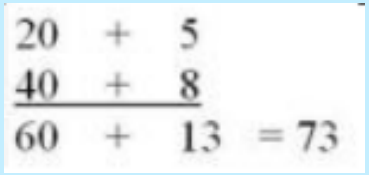
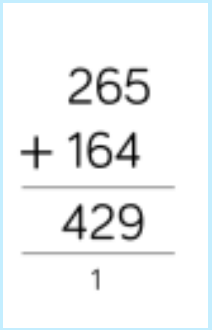
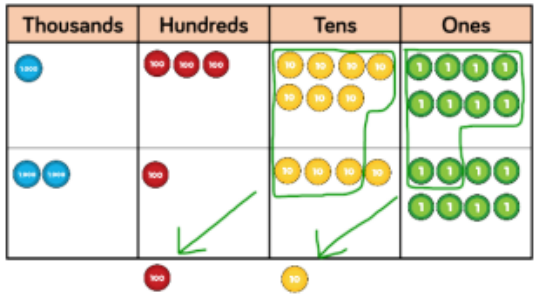
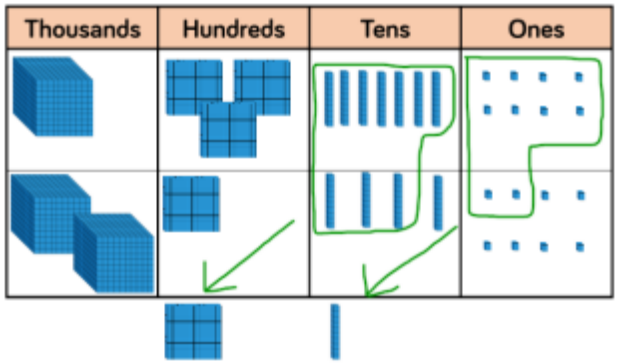
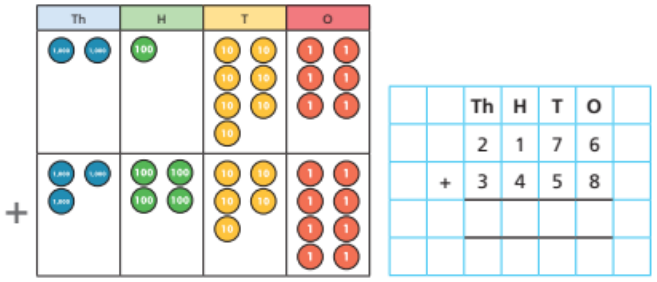
1
1
+
10
10
10
10
1
1
1

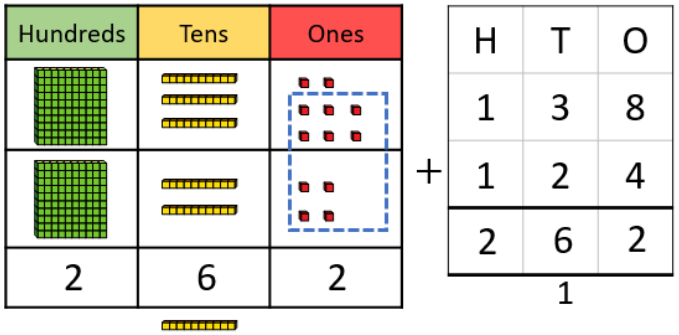
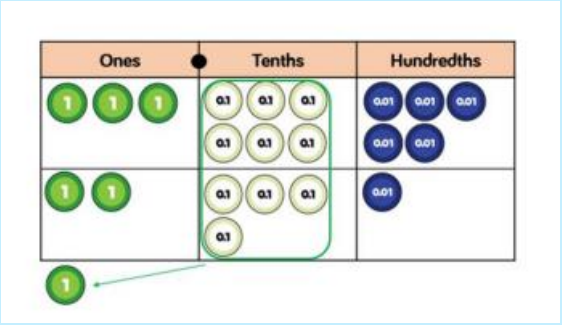
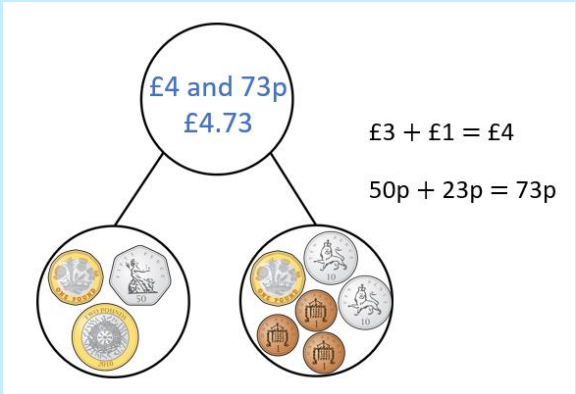
Year 3 Addition

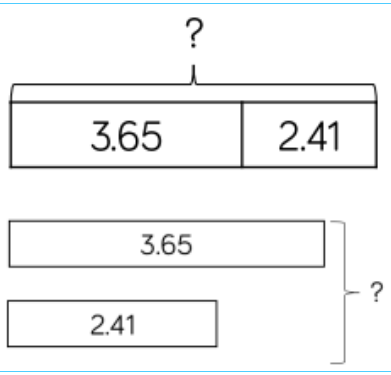
Objective	Concrete	Pictorial	Abstract
Add numbers of up to 3 digits **(Not crossing 10's/100's)**	Base 10 and counters can be used to show addition of 3 digits. 	Pictoral representations of counters, base 10 can be used as well as pictures. Children can also use methods from Year 2 to secure understanding including efficient use of a number line.    Further develop the **partitioning method** with calculations that **bridge 100:** 85 + 37 = 80 + 5 + 30 + 7 80 + 30 = 110 5 + 7 = 12 110 + 12 = 122 85 + 37 = 122 The partitioning method can also be used with three-digit numbers.	Introduce expanded method to ensure children are secure with the place value. Once secure, introduce column addition with no crossing 10's. Initially use calculations where it has not been necessary to bridge across the tens or hundreds: **63 + 32 = 95** 60 + 3 + 30 + 2 90 + 5 = 95 'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together. Recombine to give the answer.' Then… 6 3 + 3 2 5 (3 + 2) + 9 0 (60 + 30) 9 5 Add the least significant digits (units) together first and then the tens in preparation for the formal written method.   Children can problem solve and find the missing numbers.

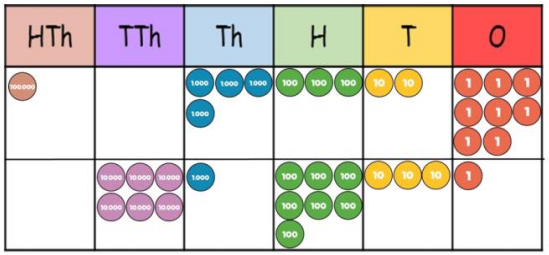
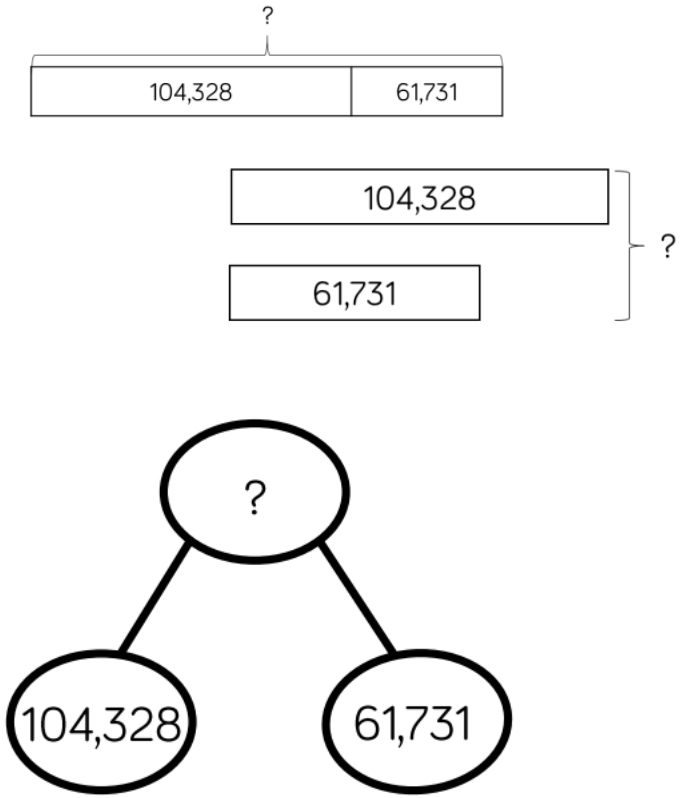
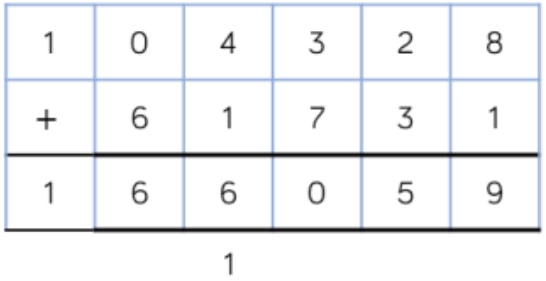
Add numbers of up to 3 digits (Crossing 10's and 100's)	Use base 10 and counters to show the children place value and regrouping   	Use pictorial representations of the base 10/ counters to show addition using regoruping. This can also demonstrate the expanded written form by adding H, T and O's.   	Formal method of expanded addition to be taught first to ensure the children understand the place value of each digit. Once children are confident then children can move onto formal column method with carrying.  

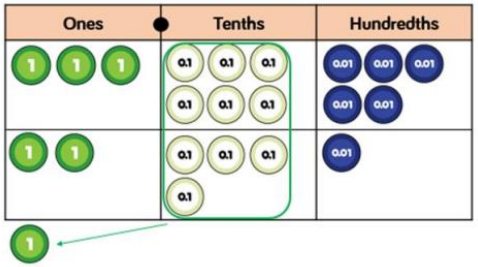
Year 4 Addition

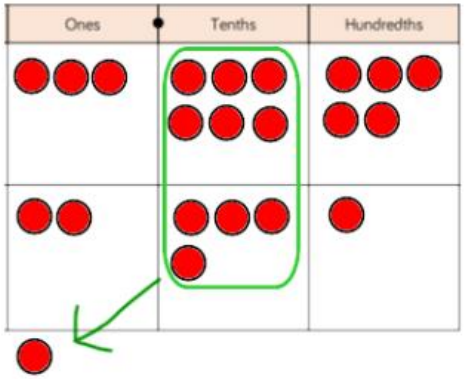
Objective	Concrete	Pictorial	Abstract
Add numbers of up to 4 digits **Exchanging/ crossing 10's** N/B Children will need to recap steps from Year 3 to add without crossing 10's to ensure method is correctly followed.	Counters and Base 10 to be used to show the children how to exchange and cross 10's/100's 	Pictorial representations can be shown and worked through to secure children's place value understanding of exchanging with larger numbers. Empty number lines are also helpful for those children who need a prompt to break down numbers into manageable chunks.  	Children to use a formal written method to add. Children can use expanded method (See Year 3) before this final step to ensure they are secure. **176 + 147 = 323** 176 + 147 13 (7 + 6) + 110 (70 + 40) 200 (100 + 100) 323 **Add 4-digit numbers** No exchange 5162 +3427 8589 Starting with the ones, add each column in turn. One exchange 5162 +3497 8659 1 Starting with the ones, add each column in turn. When adding 6 tens + 9 tens = 15 tens = 1 hundred + 5 tens Place 1 hundred under the hundreds answer and 5 tens in the answer. Multiple exchanges 5864 +3497 9361 111 Starting with the ones, add each column in turn. Exchange tens, hundreds and/ or thousands as required.

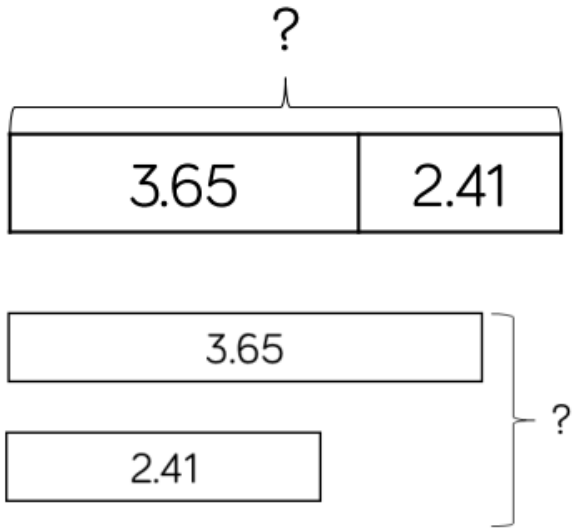
		Mr Rose earns £124 on Monday. He earns £138 on Tuesday. How much does he earn in total? £262 	Who has got each question correct? Tick your answer. a) Nijah H T O 4 4 5 + 3 4 8 78 1 3 Scott H T O 4 4 5 + 3 4 8 7 9 3 1 b) Nijah Th H T O 4 8 2 6 + 1 7 8 6 6 0 6 1 1 Scott Th H T O 4 8 2 6 + 1 7 8 5 0 0 4 1 1 1
Add decimals with 2 decimal places, including money	Use place value counters to add 2 decimals.  Money can also be used to show practical application to real life.	Children can use pictures of money to recognise the place value and decimal notation of money. Bar models are also useful in showing a whole amount. 	Children are to use formal written method to add decimals, ensuring that they know that the decimal point does not move. 3.65 + 2.41 6.06 1

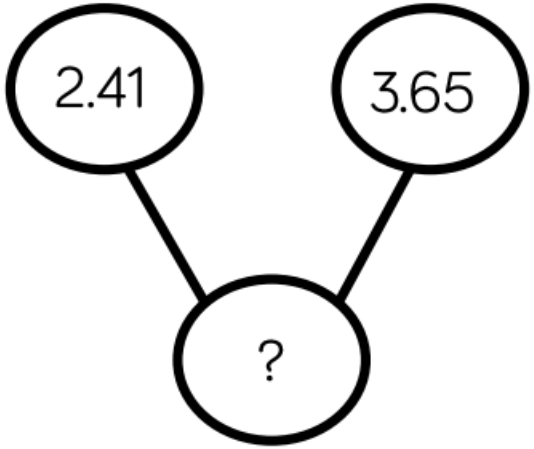
		 ? 3.65 2.41 3.65 2.41 ?		

Year 5/6 Addition			
Objective	**Concrete**	**Pictorial**	**Abstract**
add whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	Place value counters can be used to improve understanding of place value and crossing tens/hundreds etc. 	Bar models/part wholes can be used to pictorially represent the numbers alongside the abstract/concrete methods. 	At this stage, children should be encouraged to work in the abstract but the concrete and pictorial methods can be shown alongside to ensure children are confident with place value and the concepts of addition. 
Add with up to 3 decimal places	Use double sided counters and place value counters to help support the children.	Bar models and part wholes can be used to support the abstract to help give problems context.	Ensure children have experience of adding decimal places. This includes putting it into context when adding money and other measures.







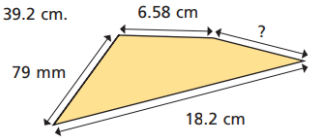


$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

Here is an irregular quadrilateral.

The perimeter of the shape is 39.2 cm.

Find the unknown length.



6·52 cm

Annie buys a book and pays with a £20 note.
She receives £6.55 change.

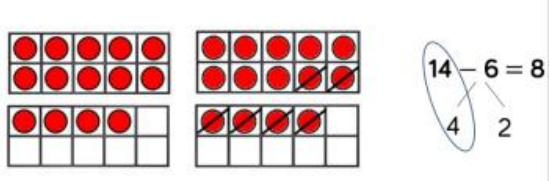
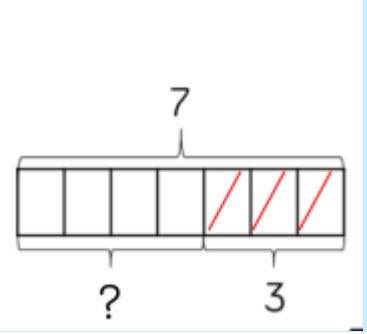
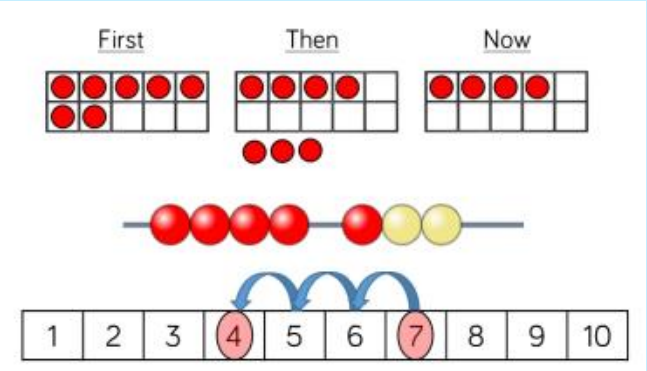
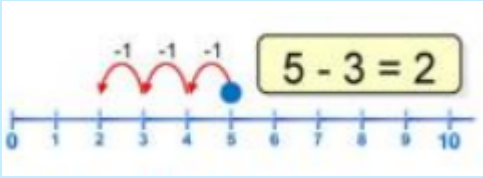
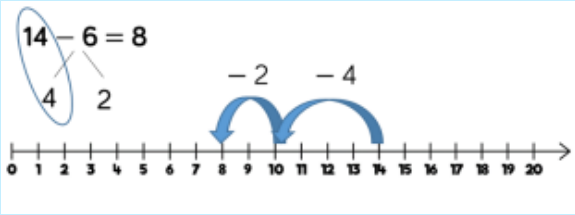
Then, Annie goes to the bakery and buys bread with a £5 note. She receives £3.30 change.

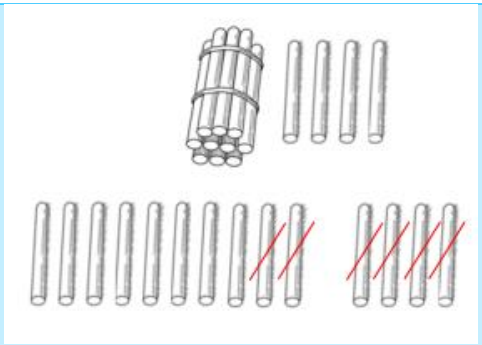
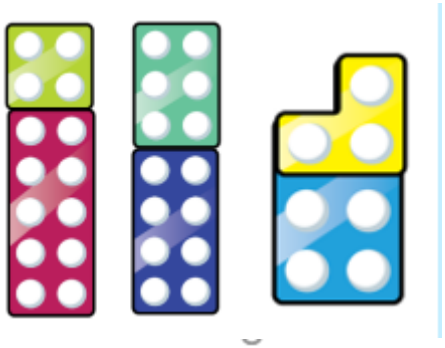
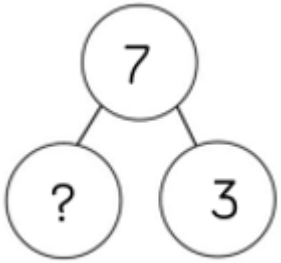
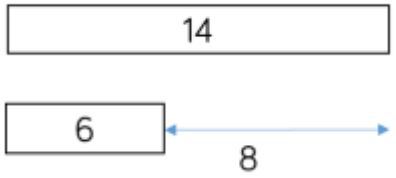
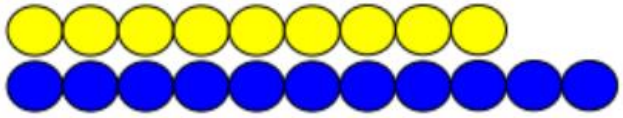
What is the total cost of the book and the bread?

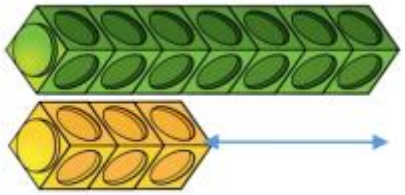
£ 15·15

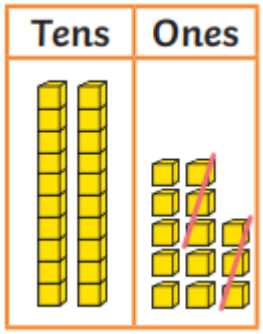
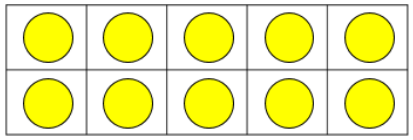
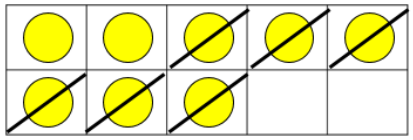
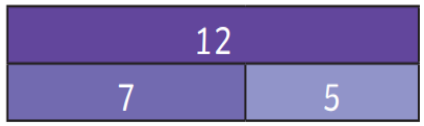
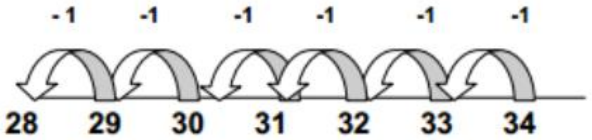
SUBTRACTION

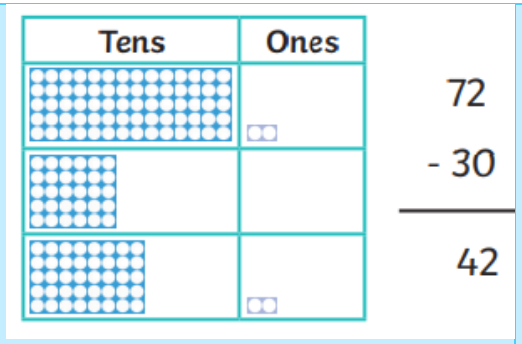
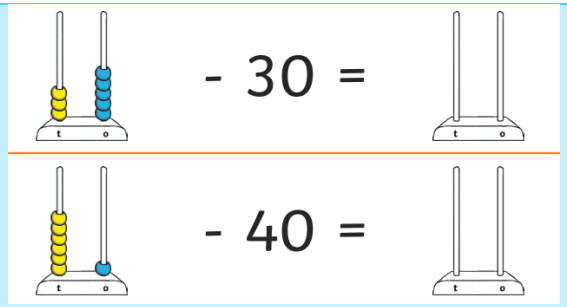
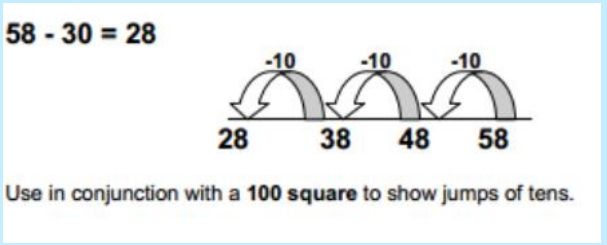
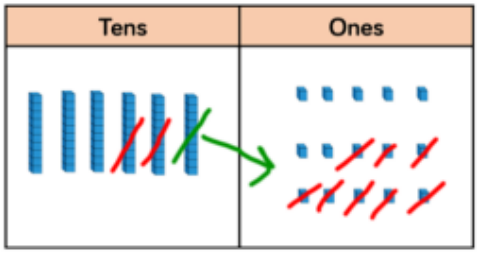
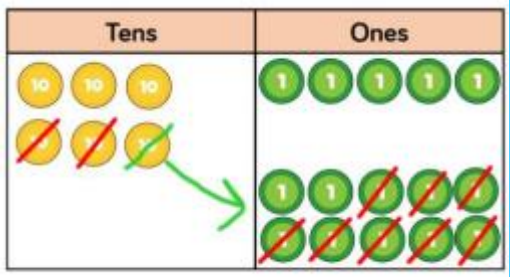
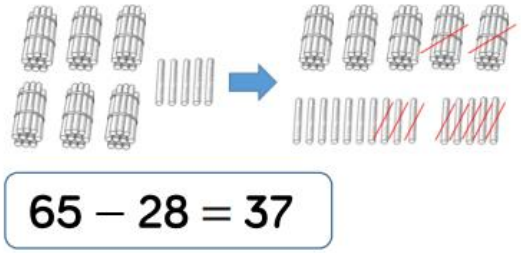
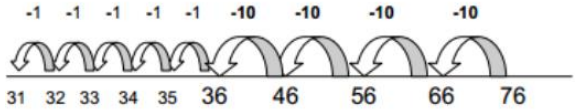
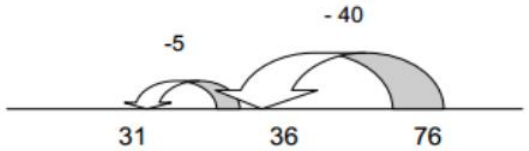
Year 1 Subtraction

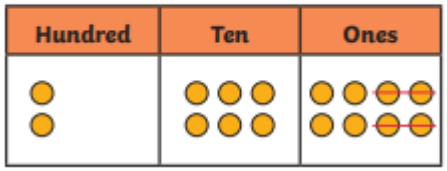
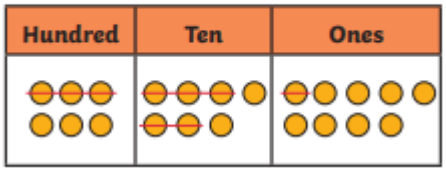
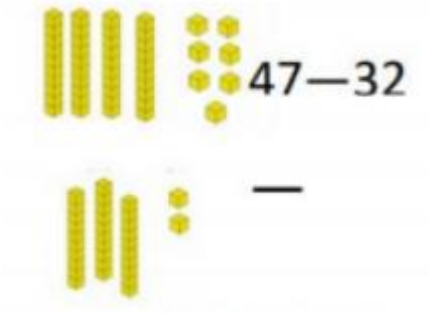
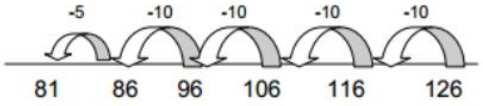
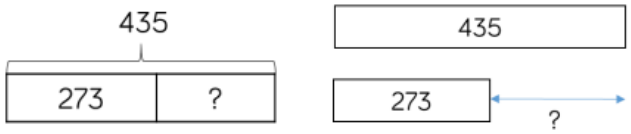
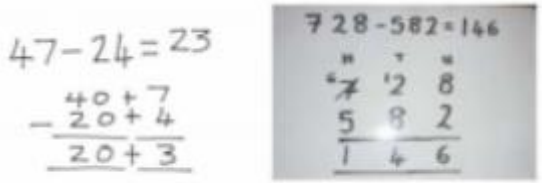
Objective	Concrete	Pictorial	Abstract
Subtract 1 and 2 digit numbers from up to 20. **(Start within 10 and once children are secure move to within 20)** **Counting back**	Use part wholes, cubes, numicon, bead strings and 10's frames to subtract children can physically remove items to show subtraction and counting back. 	Use pictures, bar models and number tracks to show subtraction by counting back.     	Children can use number lines as a formal method. Also use mental skills to solve subtraction within 20. **6 – 2 = 4** **9 – 5 = 4** 'Put your finger on number nine. Count back five.' **14 – 5 = 9**  

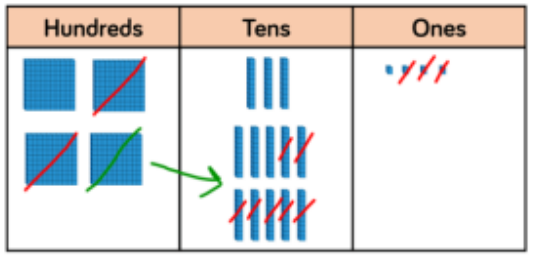
			
Finding the difference	Use part wholes, numicon, counters and cubes to model counting the difference as a way to subtract. 14-6=8 use numicon the count the difference.  	Bar models show how to count the difference  **11 – 9 = 2**  The **difference between** nine and eleven is two.	Use missing number sentences to help model finding the difference 7-_=4 12-_=8

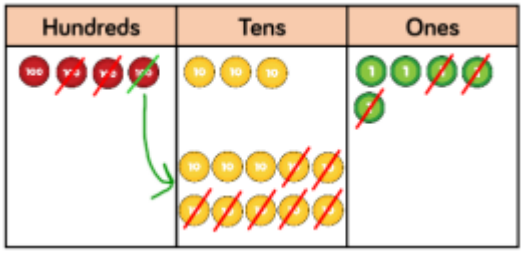
			
Make explicit links between addition and subtraction and use inverse when children are confident with basic skills to check answers.			

Year 2 Subtraction			
Objective	**Concrete**	**Pictorial**	**Abstract**
Subtracting 1 digit from 2 digit	Use base 10, 10's frames, numicon and counters to subtract single digits.  18 – 6 = ☐  	Use bar models and pictures to help understand subtraction  7 + 5 = 12 12 – 5 = 7 12 – 7 = 5 	Children can use empty number lines to subtract. Children can also use mental methods to subtract. 37 – 1 = 36 37 – 2 = 35 37 – 3 = 34  Counting back using an **empty number line** within 100, in ones… **34 - 6 = 28**  54-6= 87-9=
Subtracting in 10's	Use numicon, place value counters and base 10 to model that the ones do not change.	Use pictorial representations of taking away groups of 10 to show that the ones don't change.	Use empty number lines to take away.

			Once children are secure with place value introduce mental strategies to solve subtracting in muliples of 10.  45-20= 98-40=
Subtracting using partitioning	Children can use base 10, bead strings and counters to show counting back.  	Use pictorial representations to show partioning and counting back.  65 – 28 = 37	Children to use number line to help support subtraction. **Subtraction, using partitioning, on an empty number line:** **76 – 45 = 31**  Use in conjunction with a **100 square** to show jumps of tens and ones. Once children are confident- use more efficient jumps. **76 – 45 = 31**  76 – 40 – 5 = 31 Use in conjunction with a **100 square** to show jumps of tens and ones. 65-29= 71-37=

Year 3 Subtraction			
Objective	**Concrete**	**Pictorial**	**Abstract**
Subtract up to 3 digits using a formal method	Start with no exchanging and borrowing so the children are secure on what it means to subtract. Not crossing 10s 268 – 4 = 264  Not crossing 679 – 351 = 328   Use base 10 and counters to show methods and borrowing/exchanging.	Use number lines to show counting back in small manageable jumps. **126 – 45 = 81**  Use a **200 grid** to support counting back in tens and bridging 100  	Children are to start with expanded form so they understand the value of the digits. Once mastered move onto the more formal column subtraction.  Children are to be introduced to the formal subtraction method without borrowing to begin with. Once secure children can then move onto borrowing. (Start with smaller numbers to master the technique) 7 8 -2 3 5 5 **73 – 27 = 46** 6 13 ~~7~~ ~~3~~ - 2 7 4 6

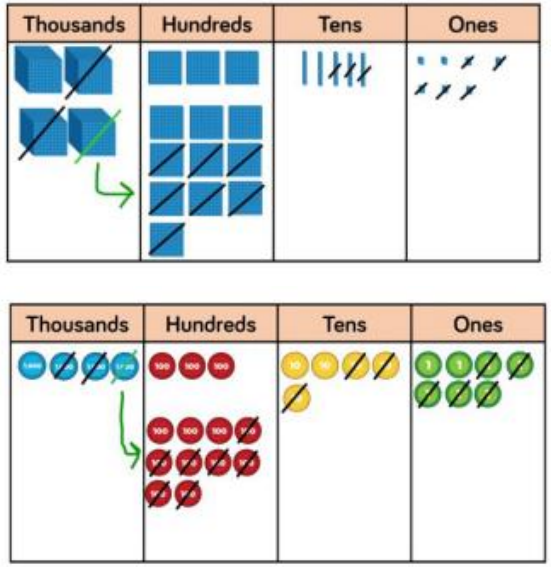
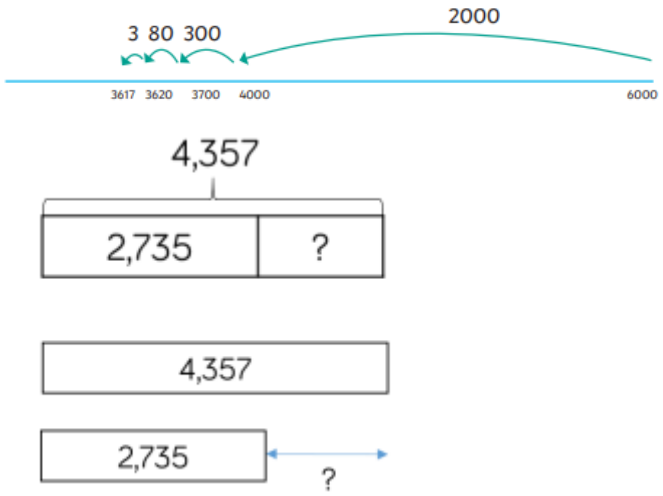


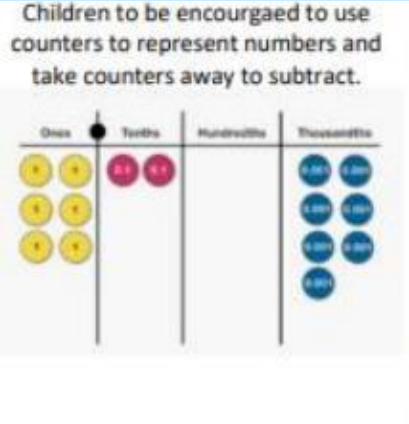
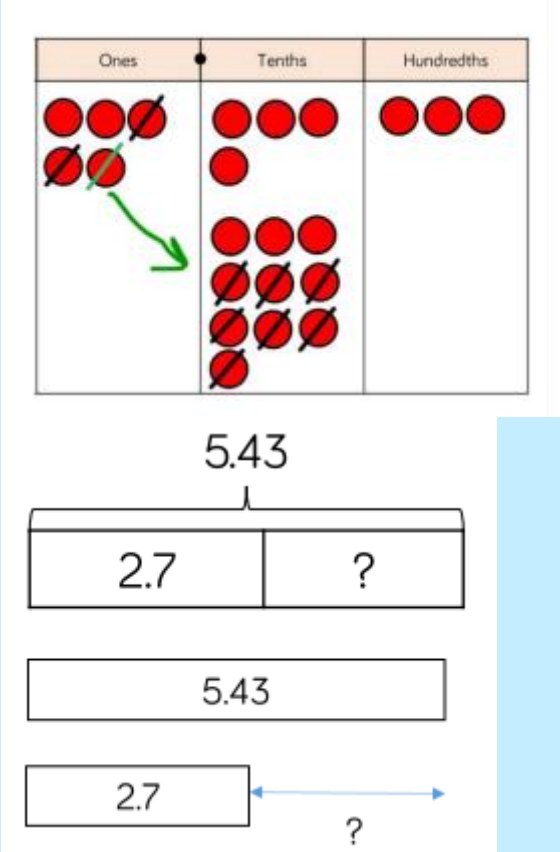
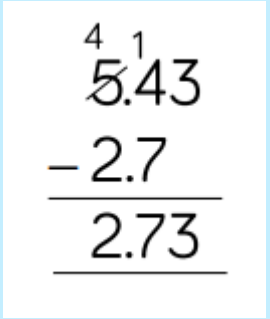


235 – 127 = 108

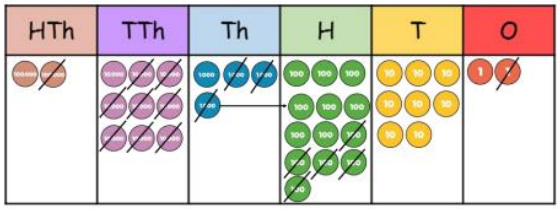
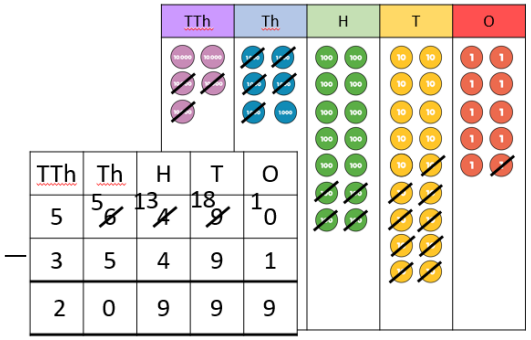
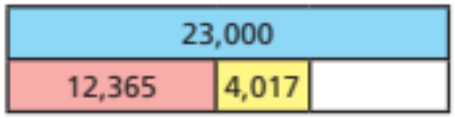
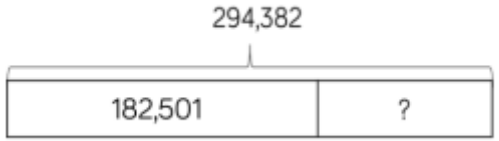
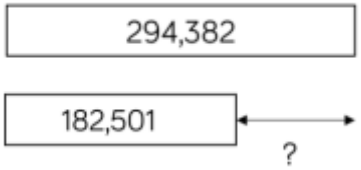
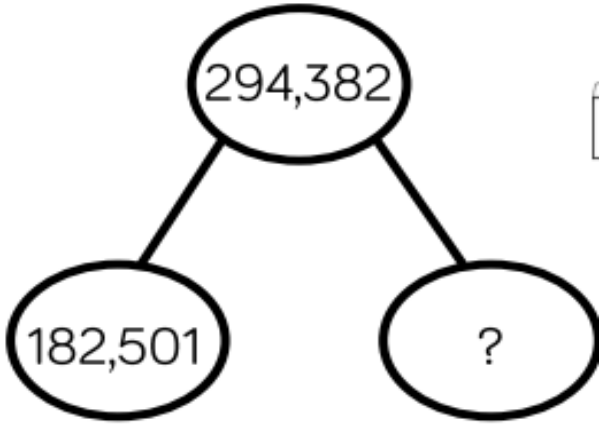
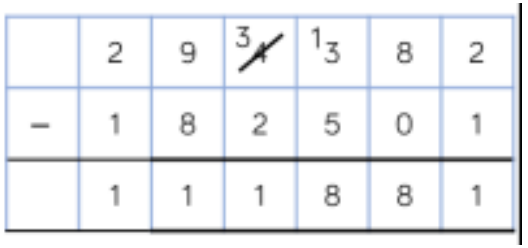
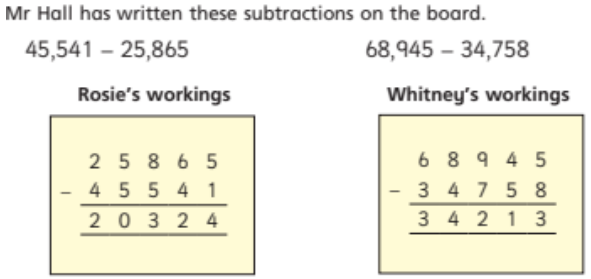
$$\begin{array}{r} {}^{2}\,{}^{15} \\ 2\,\not{3}\,\not{5} \\ -\;1\,2\,7 \\ \hline 1\,0\,8 \end{array}$$

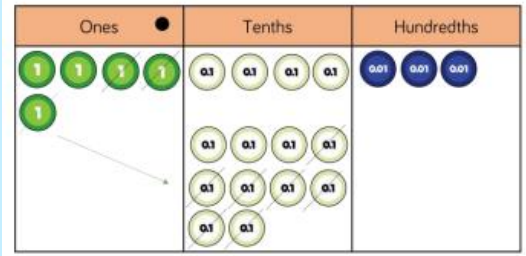
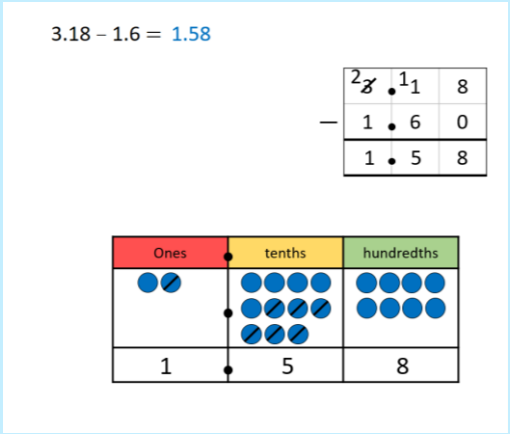
Year 4 Subtraction

Objective	Concrete	Pictorial	Abstract
Subtract up to 4 digits using a formal method	Base 10 and place value counters can be used to subtract by taking away the objects to model the number getting smaller and understanding the value of each amount. 	Bar models and number lines can be used to subtract.  Children can use the inverse to check answers once they are secure with the method. **Using Inverse** 3476 2732 744 3476 – 744 = 2732 can be checked using 2732 + 744 = 3476 This part whole shows the inverse calculations using these three numbers. 4237 1549 2688 1549 + 2688 = 4237 | 2688 + 1549 = 4237 4237 – 1549 = 2688 | 4237 – 2688 = 1549	Recap over the previous methods in Year 3 to ensure the children are secure. Work through the methods below to ensure children are confident at each stage. **Subtract 4-digit numbers** No exchange 5789 - 3421 = 2368 Starting with the ones, subtract each column in turn. One exchange 5749 - 3471 = 2278 Starting with the ones, subtract each column in turn. When subtracting 4 tens -7 tens, exchange 1 hundred to make: 14 tens – 7 tens = 7 tens Multiple exchanges 5742 - 3476 = 2266 Starting with the ones, subtract each column in turn. Exchange tens, hundreds and/ or thousands as required.

Subtract decimals with up to 2 decimal places including money	Children can use place value counters to first create the amounts to check place value understanding and then counters can be 'taken away' to model subtraction and the number getting smaller.  	Pictures, bar models and part wholes can be used to model subtraction of decimals.  	Use column subtraction methods to model subtraction ensuring the place value of the digits are correctly placed. 

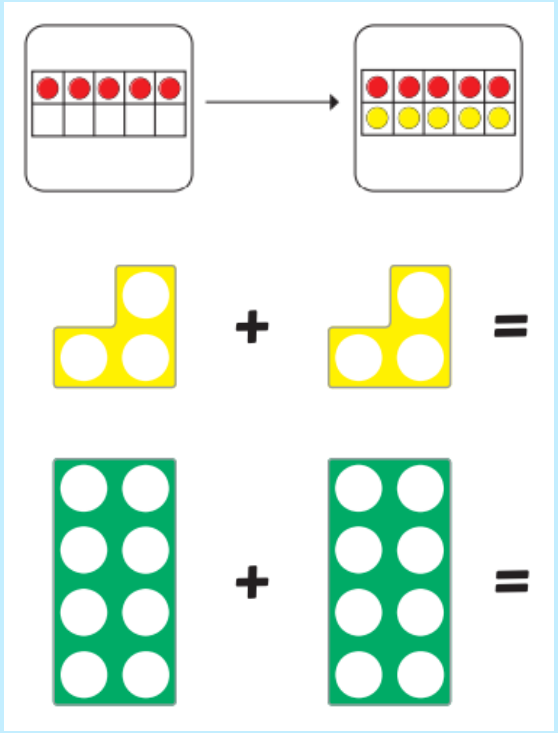
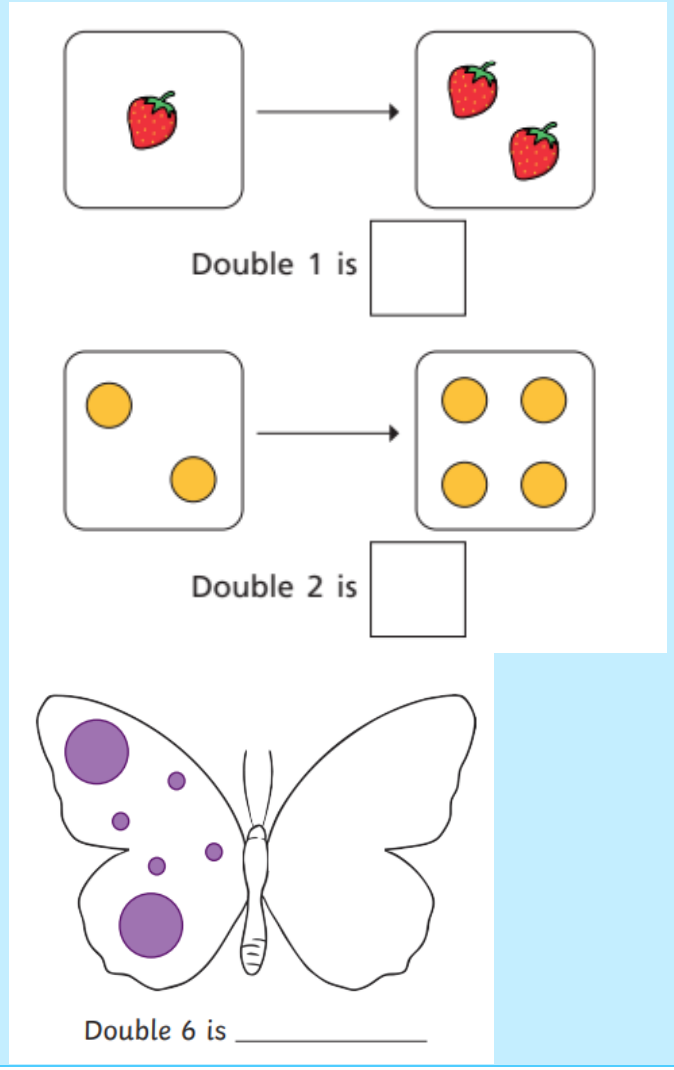
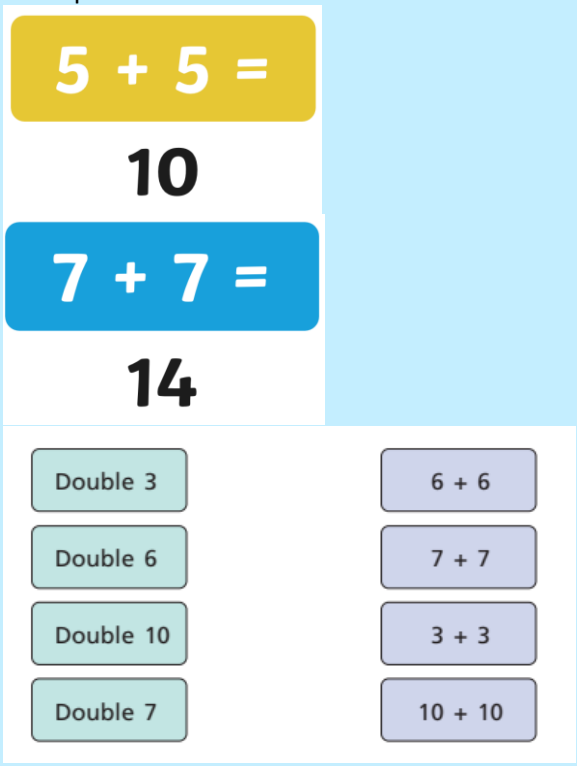
Year 5/6 Subtraction

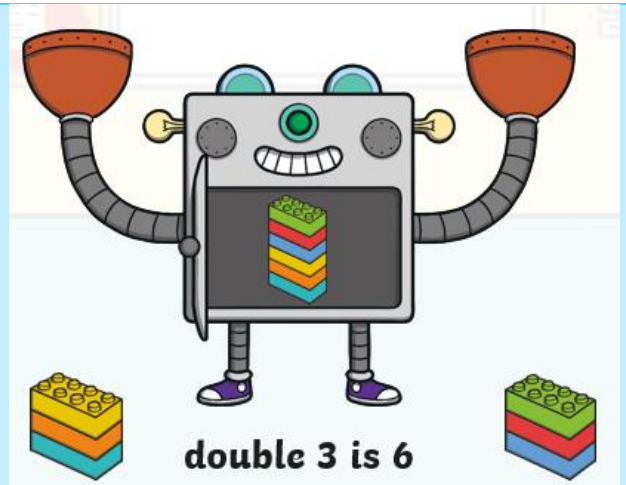
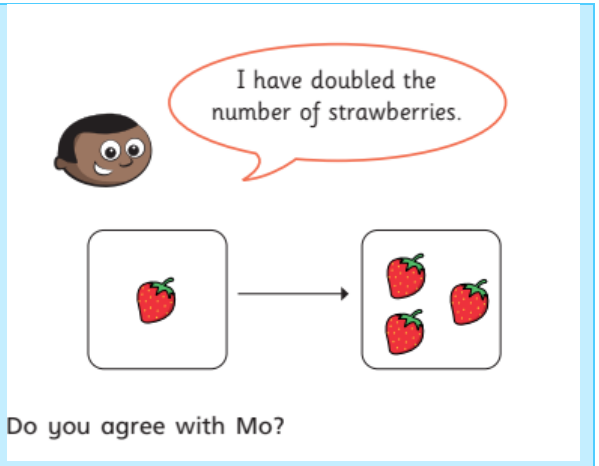
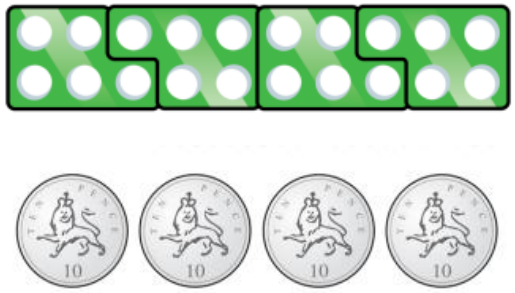
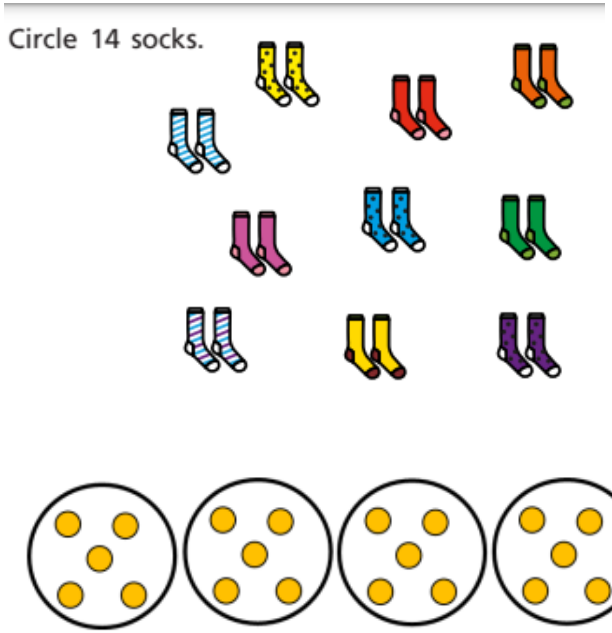
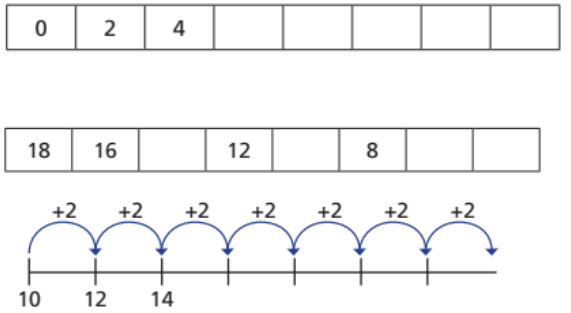
Objective	Concrete	Pictorial	Abstract
Subtract with at **least** 4 digits	Children can use place value counters in order to solve subtraction problems to show that the counters can be visually taken away.  To increase children's flexibly and fluency with number the concrete **can** be taught in line with the abstract for children to make clear connections. 56,490 − 35,491 = 20,999  	Bar models and part whole models can be used to visually show the subtraction is part of the whole amount. Pictures of place value counters can also be used where children can cross out the subtracted amounts.     	Children are to use column subtraction to solve problems. Knowledge of place value is essential.  Once children are secure they will be able to solve problems using cross curricular links and inverse to check answers.  

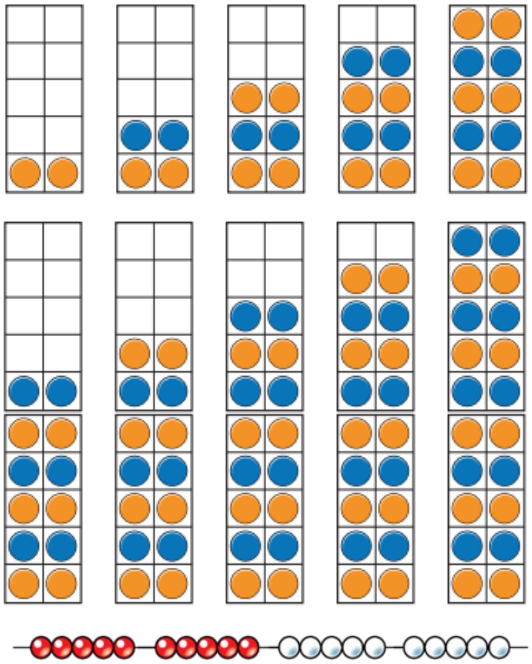
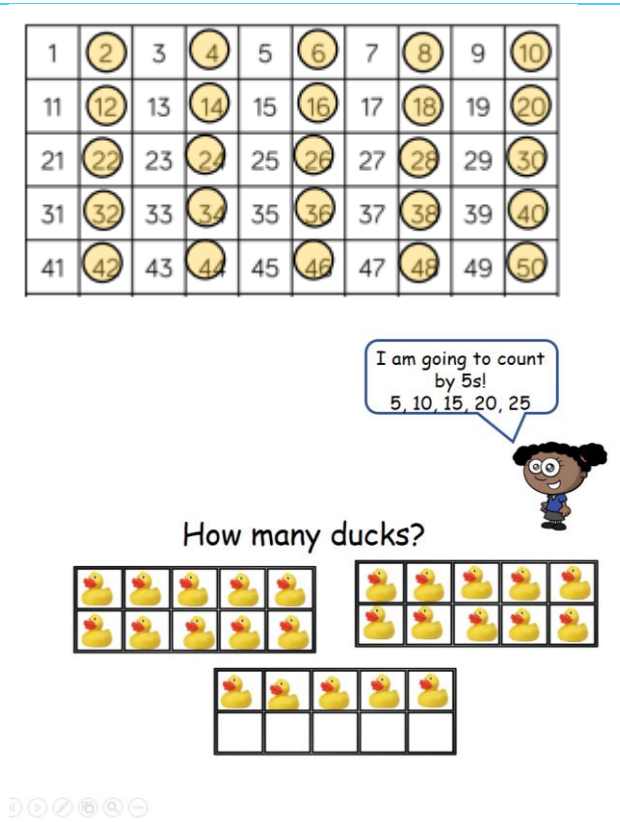
Solve increasingly complex problems with numbers with up to 3 decimal places	Children can use place value counters to take away amounts to secure understanding of the place value of each digit. 	Pictures of counters being 'crossed out' to show subtraction can be used to model subtraction. Abstract can be used in conjunction with pictorial to make connections. 3.18 – 1.6 = 1.58 	Use formal column subtraction to ensure understanding of place value and decimal point. 15 – 3.12 = 11.88 14.99 – 3.11 = Constant difference 3.11 14.99 5.43 − 2.7 = 2.73

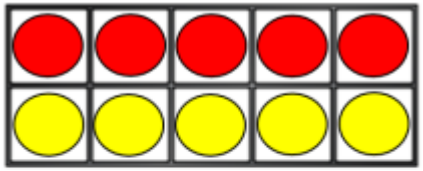
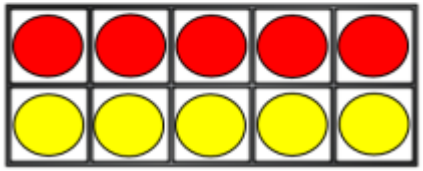
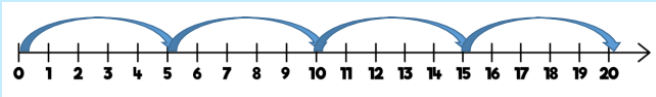
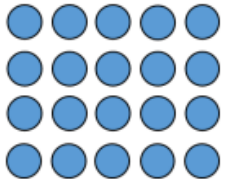
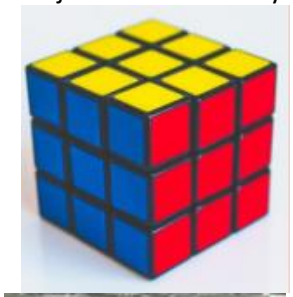
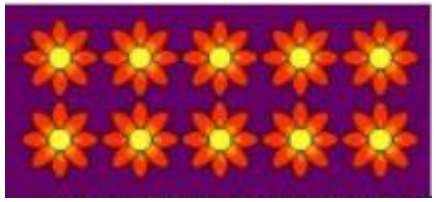
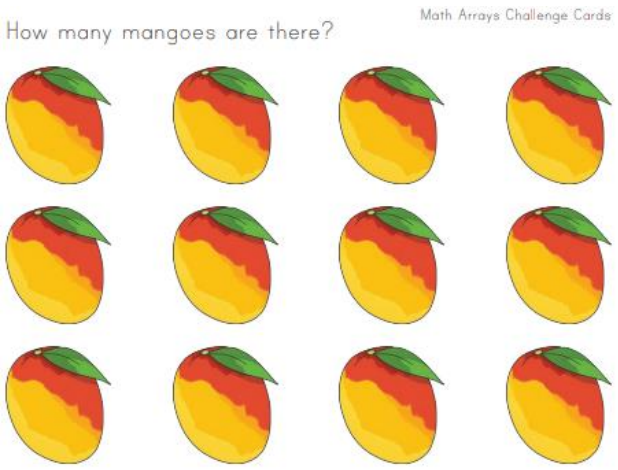
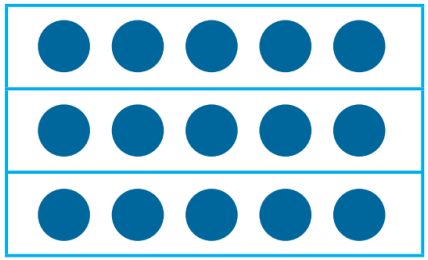
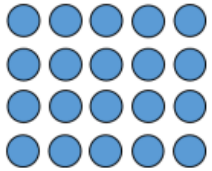
Multiplication

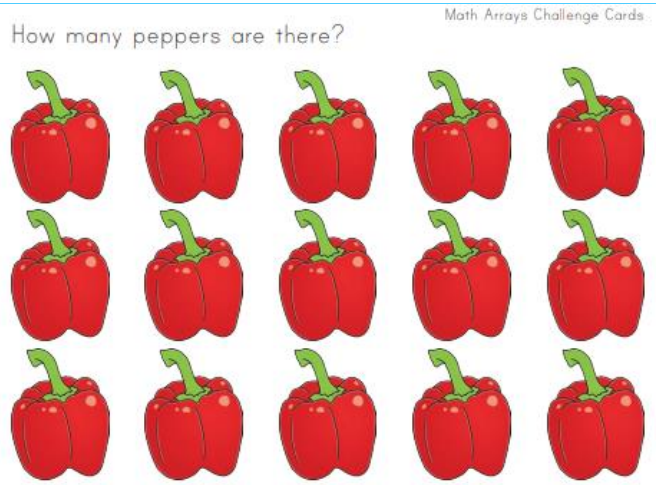
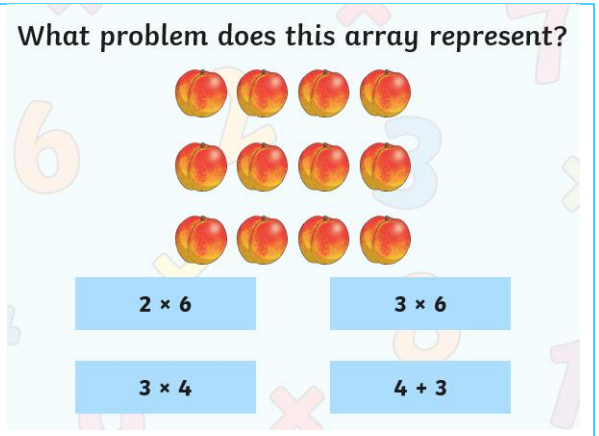
Year 1 Multiplication

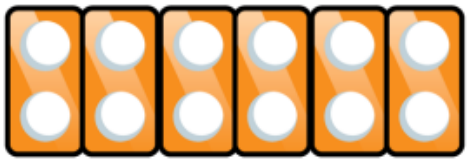
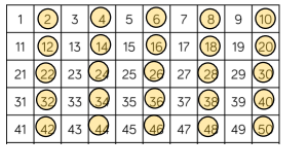
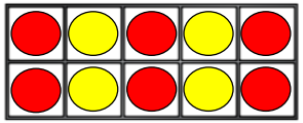
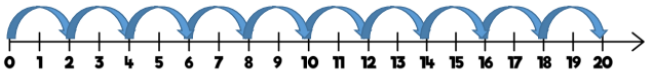
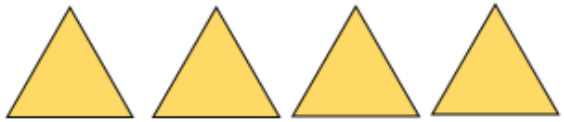
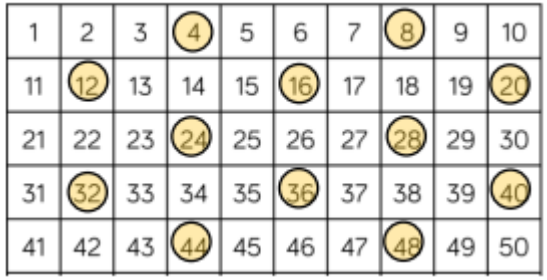
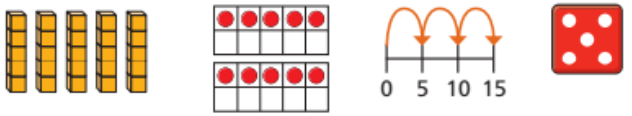
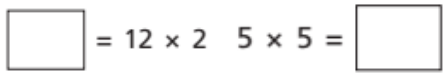
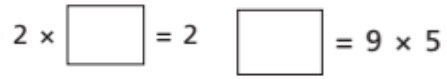
Objective	Concrete	Pictorial	Abstract
Doubling	Numicom, 10's frames and counters to be used to show doubles. 	Pictures to be used to demonstrate doubles. 	Number sentences to be used to show doubles are repeated addition. 

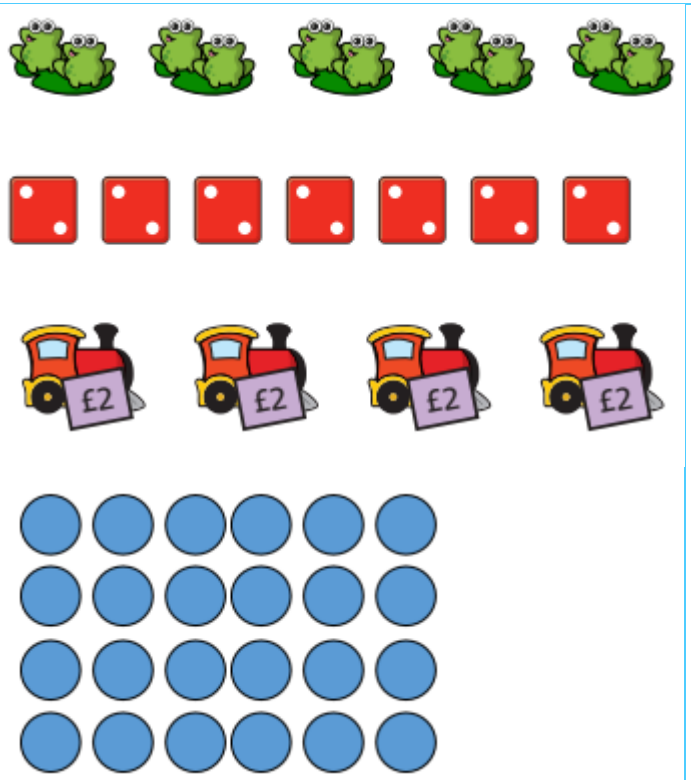
			
Counting in multiplies of 2, 5 and 10	Children can use 10's frames, bead strings, numicon, money to count in multiples of 2, 5 and 10. 	Use pictures to demonstrate counting in multiples. 	Number tracks and number lines are used to show understanding of counting up and down in multiples. Once secure children will be able to solve problems. 

			
Repeated addition	Use numicon, bead strings and counters to show repeated addition. Use abstract alongside to show the links.	Counters, number lines and pictures can be used to show repeated addition.	Use number sentences to show repeated addition $5 + 5 + 5 + 5 = 20$

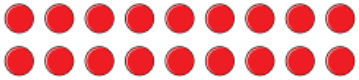
	 	  	
Using arrays	Counters and real life objects to show arrays    	Use pictures to demonstrate arrays for children to count.  	Children can draw arrays to solve problems given.  $3 \times 5 = 15$  $5 + 5 + 5 + 5 = 20$ $4 \times 5 = 20$ $5 \times 4 = 20$

		 Math Arrays Challenge Cards How many peppers are there?	 What problem does this array represent? 2 × 6 3 × 6 3 × 4 4 + 3

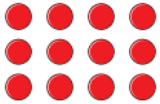
Year 2 Multiplication			
Objective	**Concrete**	**Pictorial**	**Abstract**
Counting in multiples of 2,3,4,5,10 **Counting in multiples should be taught independently.**	Use numicon, money, bead strings, counters etc to show multiples of different amounts.    	Use various pictorial representations to count in multiples.     3 6 9 12  **a)** Match the picture to the times-table fact.  3 × 5 2 × 5 1 × 5 5 × 5	  A sandwich costs £2 and a box of crayons costs £5  Jack buys 5 sandwiches and 3 boxes of crayons. How much does he spend in total? One bag holds 5 apples. How many apples do 4 bags hold?

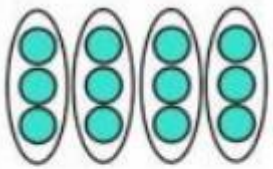
			
Multiplication is communitive	Use counters to show communitive law.	Use pictures of arrays to show the relationship of the numbers.	Number sentences and problems.

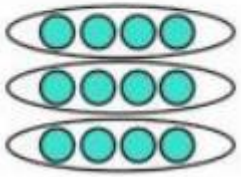
Write two multiplications for this array.



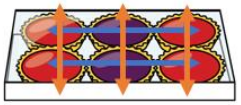
Write two additions and two multiplications for the array.

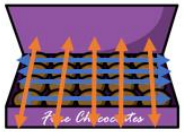




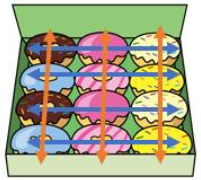


Arrays have rows and columns



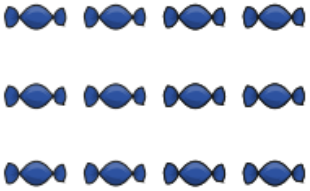


Rows



Columns

Circle each row of sweets.



How many rows are there?

There are ☐ rows.

$2 \times 10 =$ ☐

☐ $= 7 \times 10$

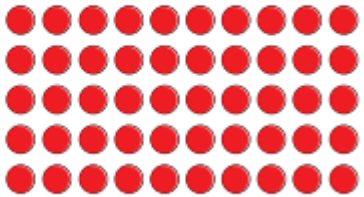
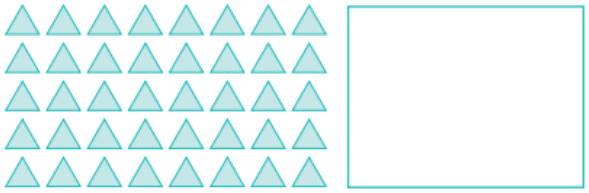
Multiplication	Array 1	Array 2
3×8		

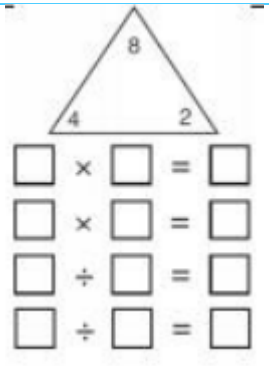
How much money does Ron have?

Complete the multiplication.

Write <, > or = to compare the calculations.

7×5	◯	5×8
6×5	◯	$4 \times 5 + 2 \times 5$
2×5	◯	$3 \times 5 - 1 \times 5$
12×2	◯	2×12

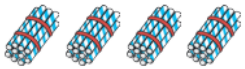
		How many pears are there?  □ + □ + □ = □ □ × □ = □ There are □ pears.	
Using the Inverse (should be taught alongside division)	Use real life arrays and counters to find the inverse. Use abstract alongside the abstract to show fact families and relationships between multiplication and division.   	Use pictures of arrays to help children understand the relationship between multiplication and division. Complete a fact family for the array.  Write two division and two multiplication facts to match this array. 	Children are to create fact families and solve problems using both multiplication and division to show the relationships between the numerals within the number sentences. Rosie has these number cards. 4 10 40 Complete the number sentences using only these numbers. □ ÷ □ = □ □ × □ = □ Are there any other ways to complete the sentences?



Match the model to the correct calculation.

8 ÷ 2 = 4

4 × 10 = 40



30 = 5 × 6

7 × ☐ = 14

12 ÷ ☐ = 2

☐ × 3 = 30

☐ ÷ 12 = 10

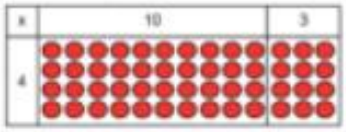
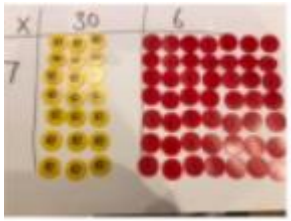
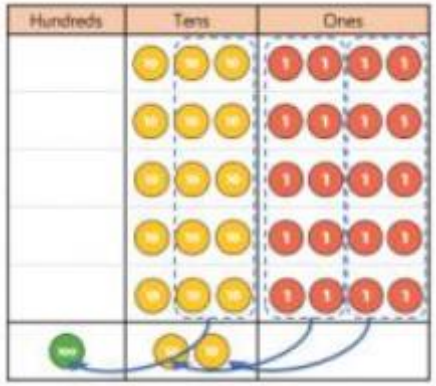
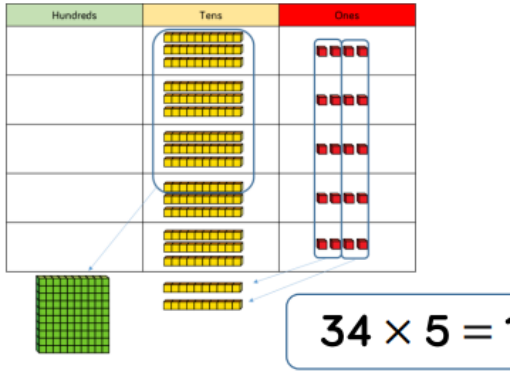
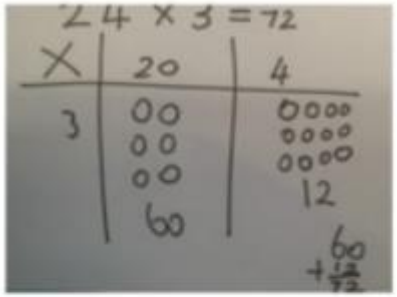
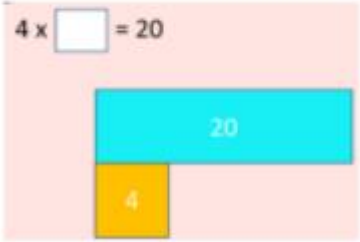
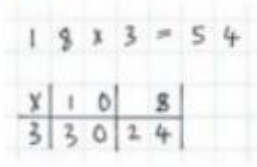
55 = 5 × ☐

5 = ☐ ÷ 4

Complete this bar model to match the calculation.

35 ÷ 5 = 7

Year 3 Multiplication

Objective	Concrete	Pictorial	Abstract
Before starting on grid method, use arrays and number lines to ensure children are secure with the idea of repeated addition. **Multiplication 2 by 1 digit.** **Grid method**	Use place value counters, counters and base 10 to visually show multiplication.     	Visually show multiplication.  Bar model are used to explore missing numbers  • Look at the problem. • Partition the large number. • Draw a grid. • Write the numbers into the grid. • Solve the calculations. • Add the answers together. • Write this number at the end of the problem to finish answering the question. *Example Answer* 22 × 4 = 88	Use grid method to understand place value of each of the digits. Start with multiplying by one digit numbers and showing the clear addition alongside the grid. TO x O  Children to add up each column to find the answer. **Grid Method** (teen number multiplied by a one- digit number): **13 x 8 = 104** 80 + 24 = 104 'Partition 13 into 10 + 3 then multiply each number by 8. Add the partial products (80 and 24) together.'

×	20		2
4	80	+	8

X	10	3
8	80	24

<table>
<tr>
<td>Multiplication of 2 by 1 digit

Expanded Form</td>
<td>See above for concrete examples of showing multiplication.

Models can be used alongside the calculations to show connections between the concrete and abstract.

<table><tr><th>Model</th><th>Calculation</th></tr><tr><td></td><td></td></tr></table></td>
<td>See above for pictorial representations of multiplication

Models can be used alongside the calculations to show connections between the pictorial and abstract.

<table><tr><th>Model</th><th>Calculation</th></tr><tr><td></td><td></td></tr></table></td>
<td>Use expanded method to undersrad
13 x 8 = 104

10 + 3
x 8
24 (3 x 8)
+ 80 (10 x 8)
104

Include an addition symbol when adding partial products.

<table><tr><td></td><td>H</td><td>T</td><td>O</td><td></td><td></td></tr><tr><td></td><td></td><td>3</td><td>4</td><td></td><td></td></tr><tr><td>×</td><td></td><td></td><td>5</td><td></td><td></td></tr><tr><td></td><td></td><td>2</td><td>0</td><td>(5 × 4)</td><td></td></tr><tr><td>+</td><td>1</td><td>5</td><td>0</td><td>(5 × 30)</td><td></td></tr><tr><td></td><td>1</td><td>7</td><td>0</td><td></td><td></td></tr></table></td>
</tr>
<tr>
<td>Multiplication of 2 by 1 digit

Short Multiplication</td>
<td>See above for concrete examples of showing multiplication.

Models can be used alongside the calculations to show connections between the concrete and abstract.

<table><tr><th>Model</th><th>Calculation</th></tr><tr><td></td><td></td></tr></table></td>
<td>See above for pictorial representations of multiplication

Models can be used alongside the calculations to show connections between the pictorial and abstract.

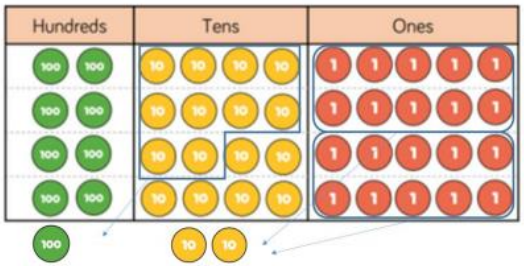
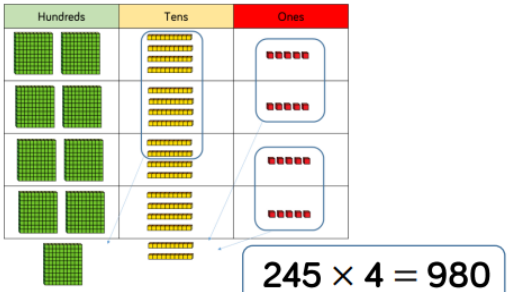
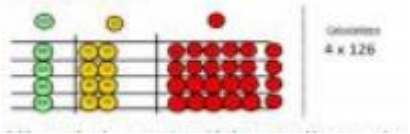
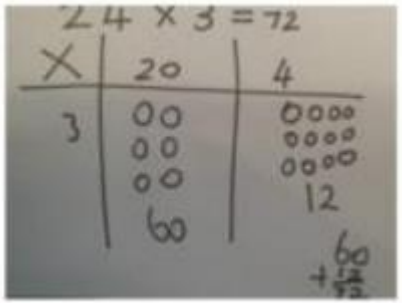
<table><tr><th>Model</th><th>Calculation</th></tr><tr><td></td><td></td></tr></table></td>
<td>Once children are secure with the expanded form, children to move onto the short column method of multiplication.

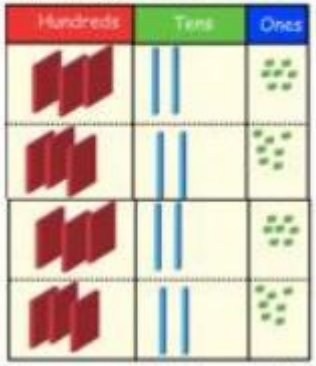
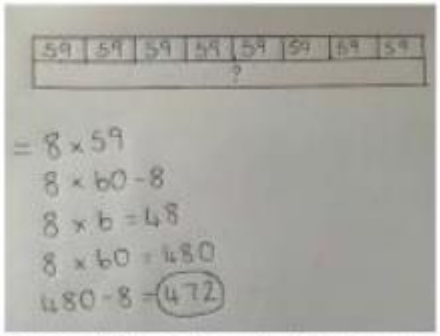
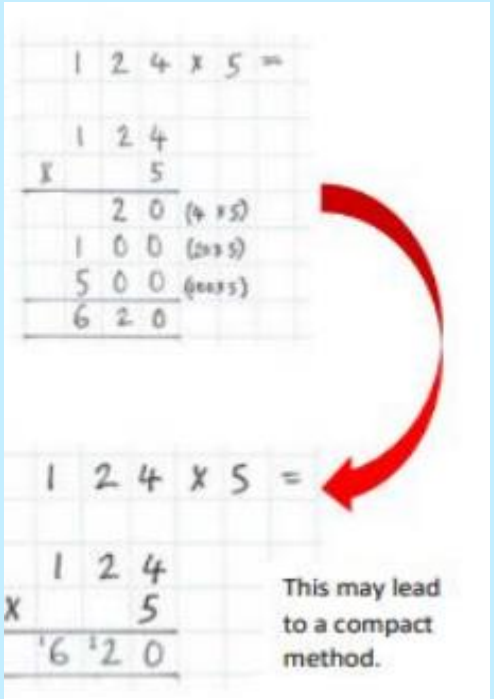
<table><tr><td></td><td>H</td><td>T</td><td>O</td><td></td></tr><tr><td></td><td></td><td>3</td><td>4</td><td></td></tr><tr><td>×</td><td></td><td></td><td>5</td><td></td></tr><tr><td></td><td>1</td><td>7</td><td>0</td><td></td></tr><tr><td></td><td>1</td><td>2</td><td></td><td></td></tr></table>
Formal short multiplication:

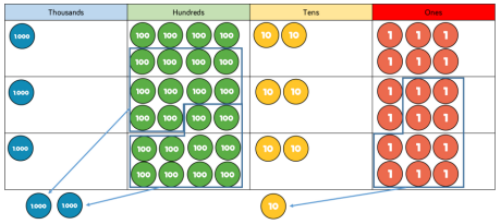
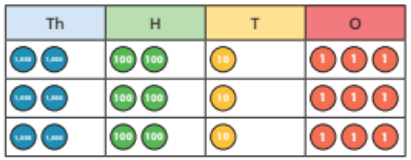
1 3
x 8
1 0 4
2

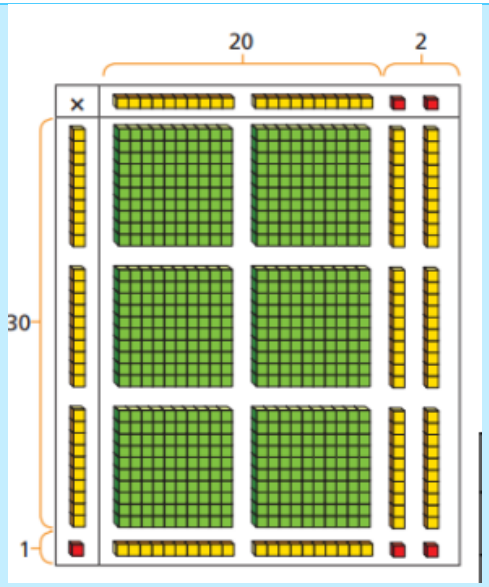
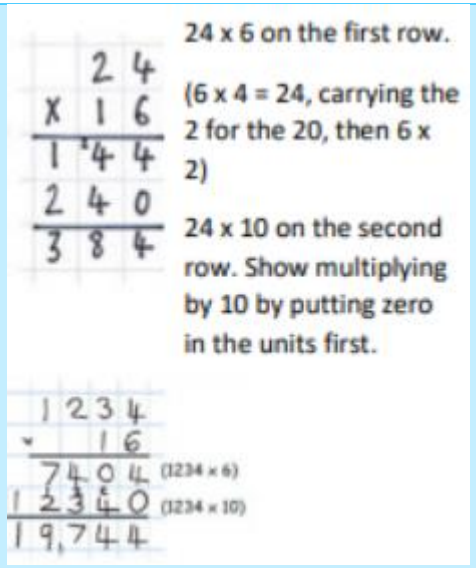
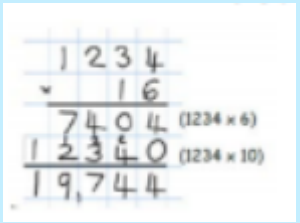
Ensure that the digit ‘carried over’ is written under the line in the correct column.

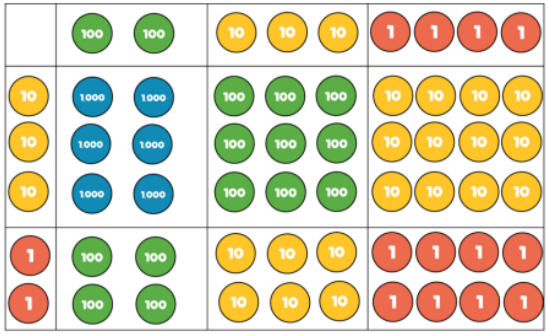
Use the language of place value to ensure understanding.</td>
</tr>
</table>

Year 4 Multiplication			
Objective	**Concrete**	**Pictorial**	**Abstract**
Multiply 2 and 3 digit numbers by 1 digit **(Recap grid method from Year 3)** **Expanded method**	Use counters and base 10 to show multiplication.   $245 \times 4 = 980$ Models can be used alongside the calculations to show connections between the concrete and abstract. Model | Calculation	Use pictorial methods to show multiplication.   Models can be used alongside the calculations to show connections between the pictorial and abstract. Model | Calculation	HTO x O  Children to add up each column to find the answer. **36 x 4 = 144** 30 + 6 X 4 24 (4 x 6 = 24) +120 (4 x 30 = 120) 144 **127 x 6 = 762** 127 x 6 42 (6x7) +120 (6x20) 600 (6x100) 762

Column/ Short multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. Models can be used alongside the calculations to show connections between the concrete and abstract. | Model	Calculation				Bar models can be used to model repeated addition.  Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods. Models can be used alongside the calculations to show connections between the pictorial and abstract. | Model	Calculation				Recap over expanded method and move onto short multiplication. Children should be secure on short multiplication method.  |	H	T	O
	2	4	5
x			4
	9	8	0
	1	2	|

Year 5/6 Multiplication			
Objective	**Concrete**	**Pictorial**	**Abstract**
4 digit multiplication by 1 digit **(Repeat steps from Year 4)**	Use base 10 and place value counters to show understanding of place value when multiplying.  	Complete the sentences to describe the multiplication.  There are ☐ ones altogether. There are ☐ tens altogether. There are ☐ hundreds altogether. There are ☐ thousands altogether. 2,213 × 3 = ☐ Models can be used alongside the calculations to show connections between the pictorial and abstract. **Model** | **Calculation**	Short multiplication method to be used. $1{,}826 \times 3 = 5{,}478$ Th H T O 1 8 2 6 × 3 5 4 7 8 2 1
Multiply 2 by 2 digit numbers **Long multiplication**	Use base 10 and place value counters to show understanding of place value when multiplying.	Use pictorial representations of the multiplications to help understanding of place value.	Children to use long multiplication method.

	 Manipulatives may still be used with the corresponding long multiplication modelled alongside Model | Calculation	 Pictures can be used alongside the calculations × | 20 | 2 30 | 600 | 60 1 | 20 | 2 Models can be used alongside the calculations to show connections between the pictorial and abstract. Model | Calculation	  | H | T | O | | | 2 | 2 | × | | 3 | 1 | | | 2 | 2 | 6 | 6 | 0 | 6 | 8 | 2 |
3 digits by 2 digits	Use base 10 and place value counters to show understanding of place value when multiplying.	Pictures can be used alongside the calculations	Long multiplication method to be used.

<table>
<tr>
<td></td>
<td>


Manipulatives may still be used with the corresponding long multiplication modelled alongside
<table>
<tr><th>Model</th><th>Calculation</th></tr>
<tr><td></td><td></td></tr>
</table>
</td>
<td>
<table>
<tr><td>×</td><td>200</td><td>30</td><td>4</td></tr>
<tr><td>30</td><td>6,000</td><td>900</td><td>120</td></tr>
<tr><td>2</td><td>400</td><td>60</td><td>8</td></tr>
</table>
Models can be used alongside the calculations to show connections between the pictorial and abstract.
<table>
<tr><th>Model</th><th>Calculation</th></tr>
<tr><td></td><td></td></tr>
</table>
</td>
<td>
$234 \times 32 = 7{,}488$
<table>
<tr><td>Th</td><td>H</td><td>T</td><td>O</td></tr>
<tr><td></td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>×</td><td></td><td>3</td><td>2</td></tr>
<tr><td></td><td>4</td><td>6</td><td>8</td></tr>
<tr><td>$^{1}7$</td><td>$^{1}0$</td><td>2</td><td>0</td></tr>
<tr><td>7</td><td>4</td><td>8</td><td>8</td></tr>
</table>
</td>
</tr>
<tr>
<td>4 by 2 digit multiplication</td>
<td>Use base 10 and place value counters to show understanding of place value when multiplying. See above

You may use place value counters to help.
<table>
<tr><th>×</th><th>1,000</th><th>200</th><th>30</th><th>4</th></tr>
<tr><td>20</td><td></td><td></td><td></td><td></td></tr>
<tr><td>6</td><td></td><td></td><td></td><td></td></tr>
</table>
</td>
<td>Models can be used alongside the calculations to show connections between the pictorial and abstract.
<table>
<tr><th>Model</th><th>Calculation</th></tr>
<tr><td></td><td></td></tr>
</table>
</td>
<td>Long multiplication method to be used.

$2{,}739 \times 28 = 76{,}692$</td>
</tr>
</table>

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2 (2)	1 (5)	9 (3)	1 (7)	2
5 (1)	4	7 (1)	8	0
7	6	6	9	2
		1		

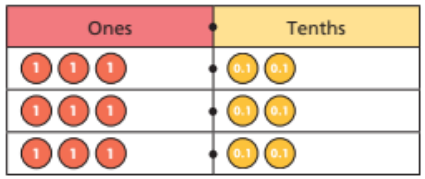
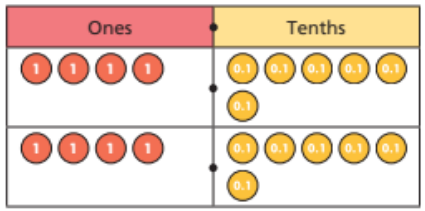
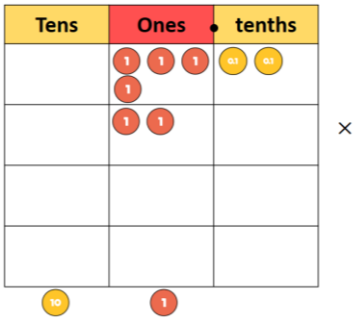
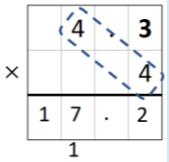
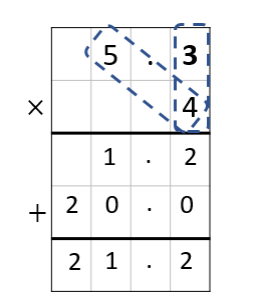
Rosie is calculating 2,541 × 42

Here is Rosie's working.

	2	5	4	1	
×			4	2	
	4_1	0	8	2	(2,541 × 2)
	8_2	0_1	6	4	(2,541 × 40)
1_1	2	1_1	4	6	

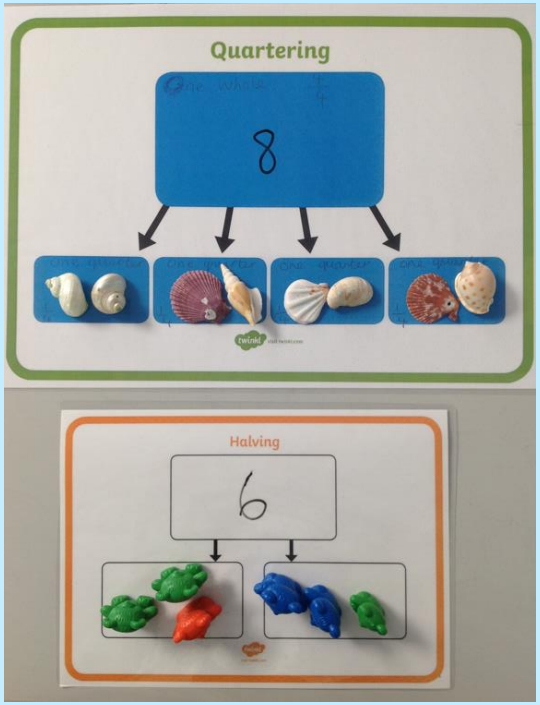
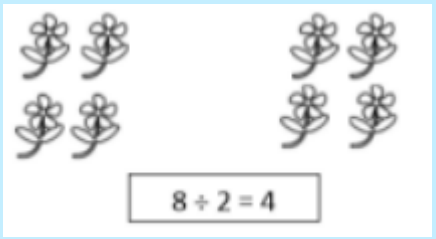
a) Rosie has made two mistakes. What are they?

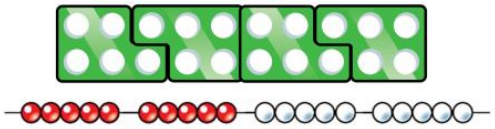
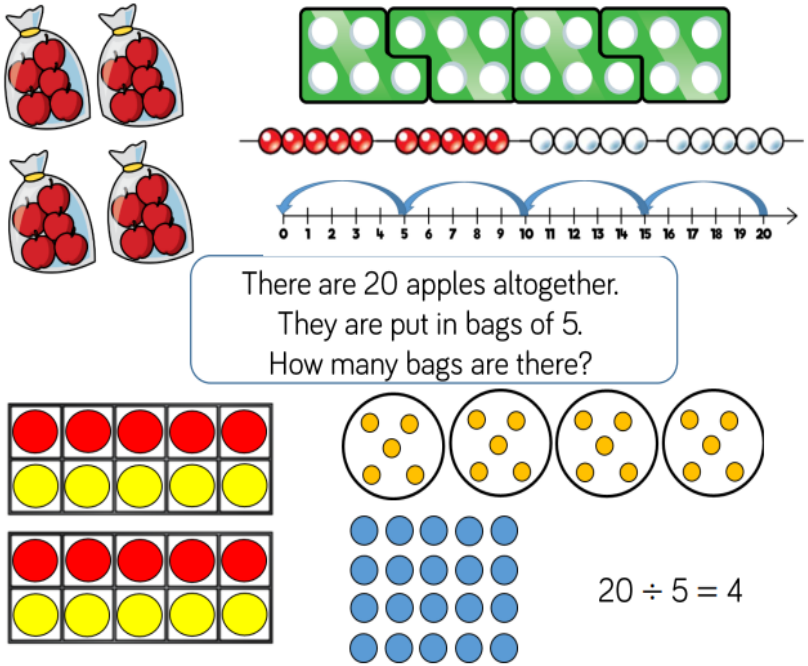
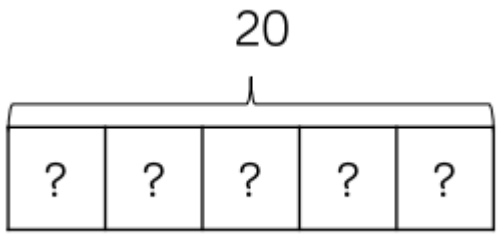
b) What is the correct answer?

Year 6 Multiplication			
Objective	**Concrete**	**Pictorial**	**Abstract**
Same objectives as Year 5			
Multiply decimals	Use place value counters to show place of each number. 3.2 × 3  4.6 × 2  	Use pictorial representations and children can draw the problems. 4.3 × 4 = 17.2   	Use formal method to solve problems.  

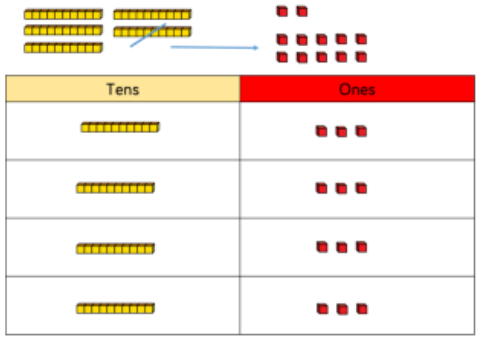
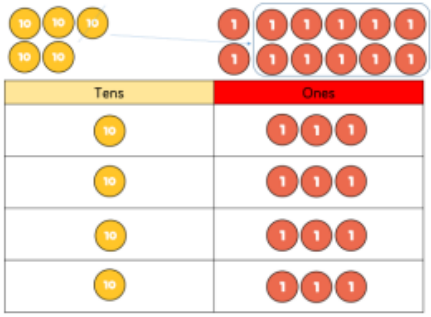
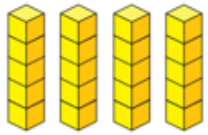
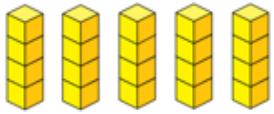
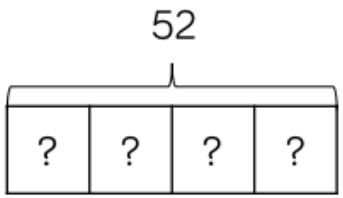
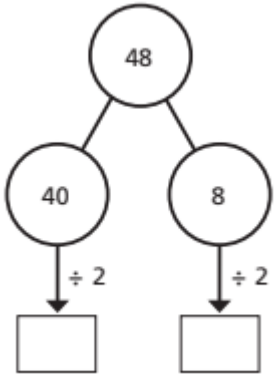
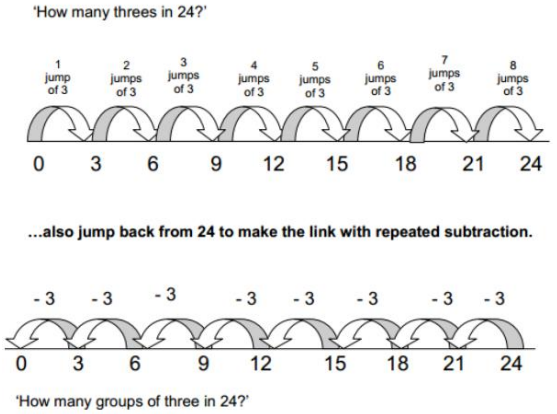
DIVISION

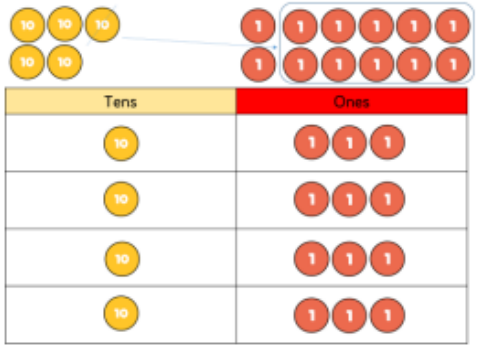
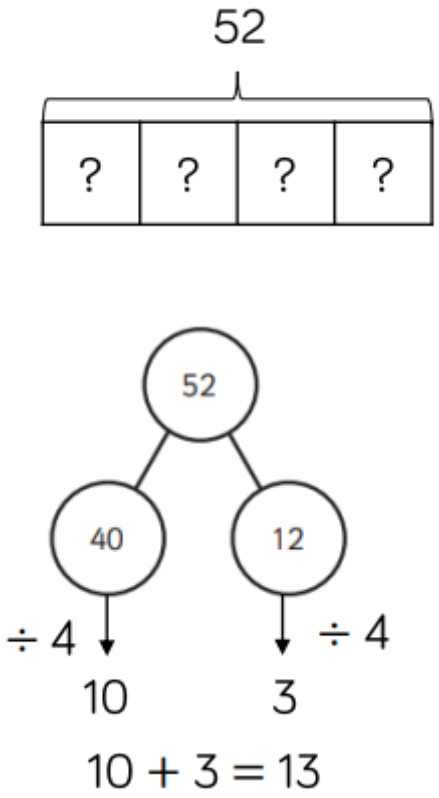
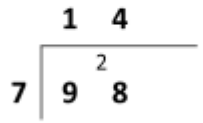
Year 1 Division

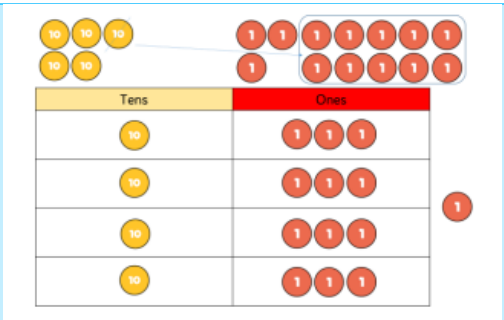
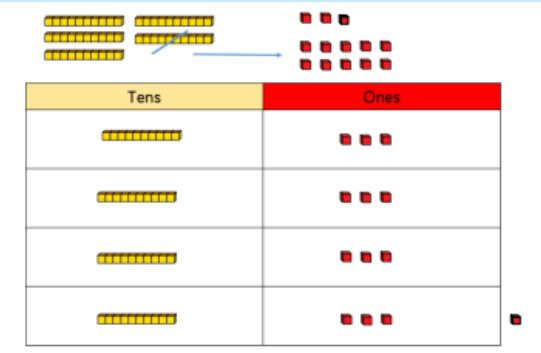
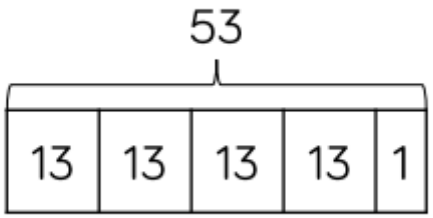
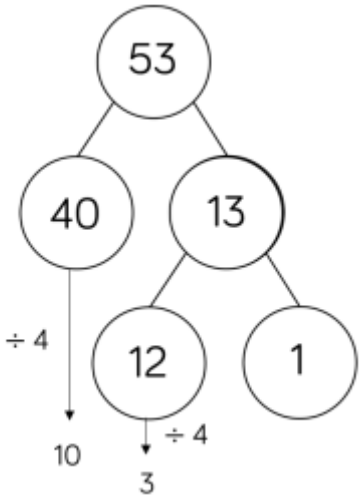
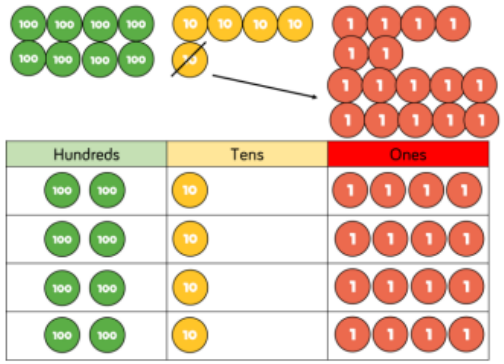
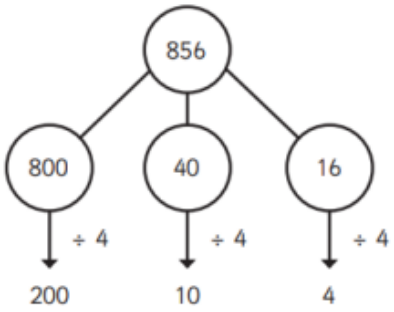
Objective	Concrete	Pictorial	Abstract
Division as sharing (linked with fractions ½ 1/4)	Children to use resources in order to share. Can be used in conjunctions with fractions. 	Children use pictures or shapes to share quantities 	Link to fractions ½ of 12= 6 ¼ of 8=2

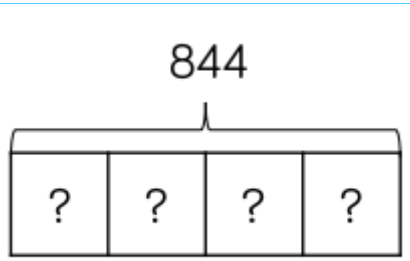
Year 2 Division			
Objective	**Concrete**	**Pictorial**	**Abstract**
Division as grouping (Link to fractions of amounts)	Use counters, numicon and bead strings to group/share objects.  	Use pictures to represent division as grouping and to help solve problems.  Bar models help children to see parts of a whole amount when sharing. 	Problems written as number sentences and word problems should be used. Links with fractions of amounts Divide 30 into 5 groups. How many are in each group? 20÷4= 5 ½ of 24= ¼ of 16=

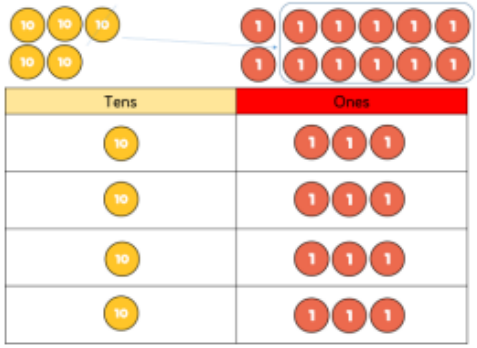
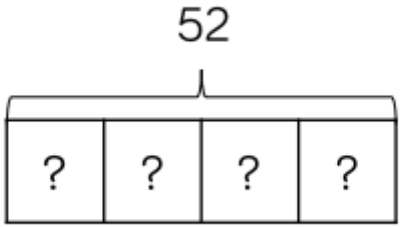
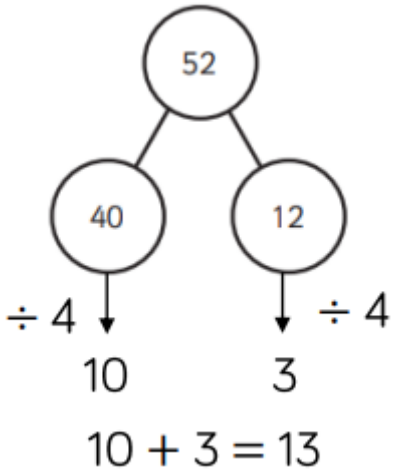
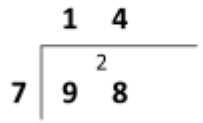
Arrays to support division- Links made with multiplication	Children can make arrays with counters, objects to help support their understanding of the links between multiplication and division.	**Using arrays to support division** **15 ÷ 5 = 3** **15 ÷ 3= 5**  How many groups of 3? How many groups of 5? 15 shared between 3 people is…? 15 shared between 5 people is…? 15 divided by 5 = 3 15 divided by 3 = 5	

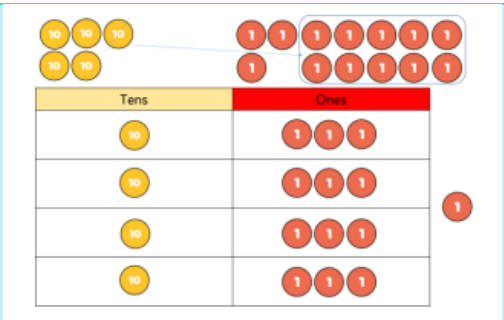
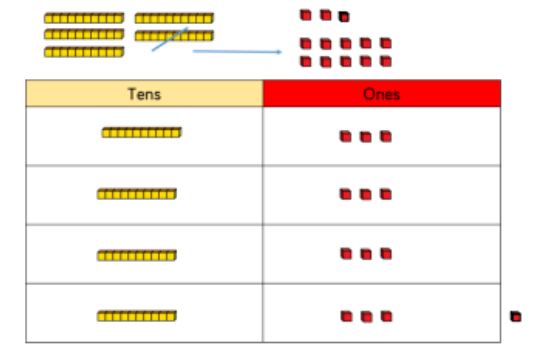
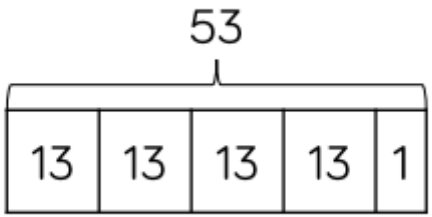
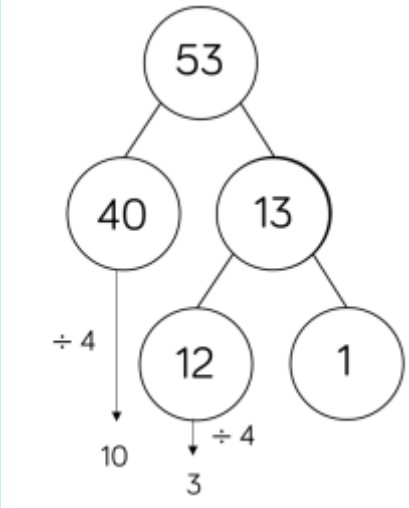
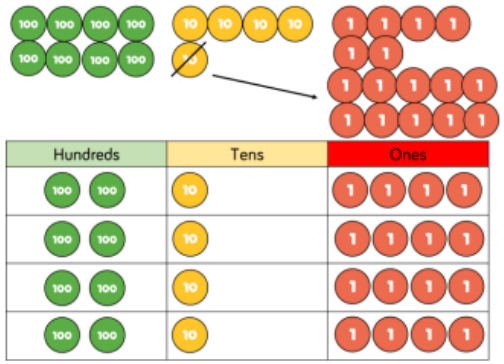
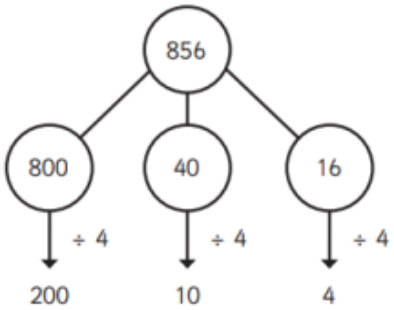
Year 3 Division			
Objective	**Concrete**	**Pictorial**	**Abstract**
Division 2 digit by one digit **NO REMAINDERS**	Use counters, cubes, objects to share into equal groups. When using base 10 and place value tokens exchanging will need to be addressed.   	Use part wholes and bar models to demonstrate understanding of division steps. $20 \div 4 = 5$ Shared into 4 equal towers  There are **5** cubes in each tower. Grouped into towers of 4  There are **5** towers of 4 cubes.   	More formally recorded methods can be introduced such as an empty number line counting forward or back in 'jumps'  Once the children have mastered this method they can move onto short division if they are confident with their times tables knowledge. **24 ÷ 3 = 8** This can also be recorded as... $3 \overline{)\,24}$ (8) 'Twenty four divided by three equals eight.' 'How many threes are there in twenty four?'

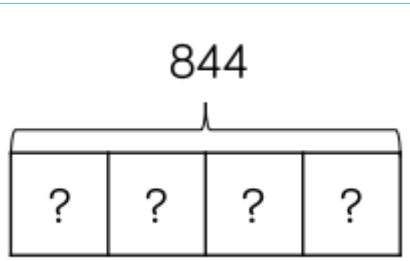
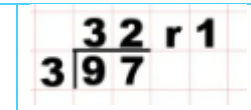
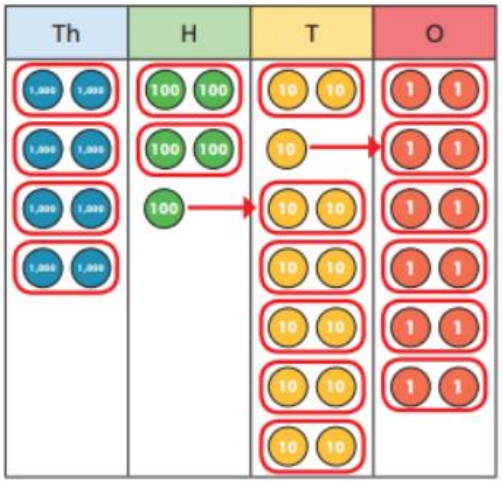
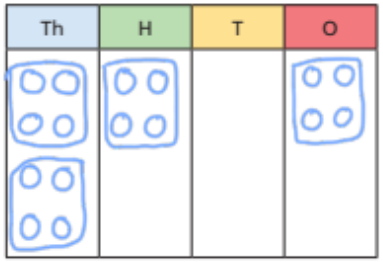
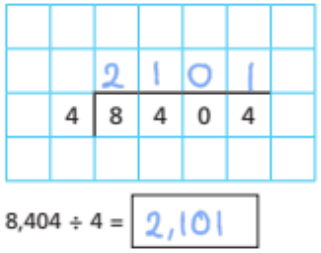
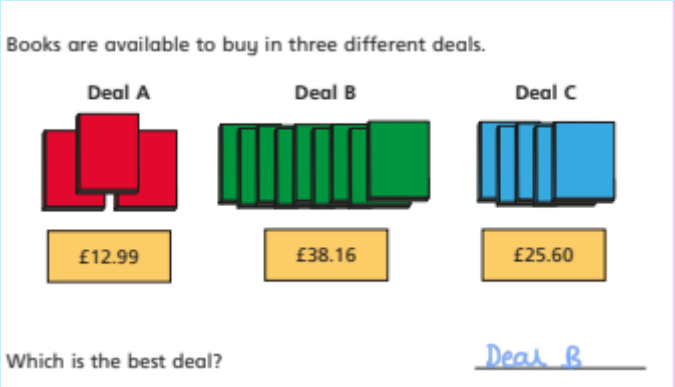
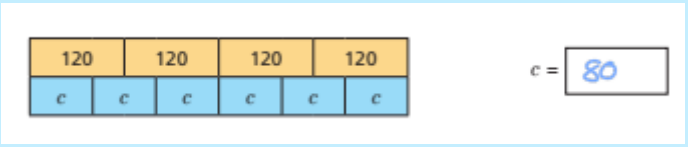
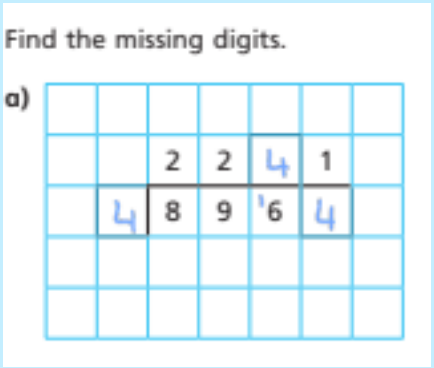
Year 4 Division			
Objective	**Concrete**	**Pictorial**	**Abstract**
Division 2 by one digit No remainders	Use counters, cubes, objects to share into equal groups. When using base 10 and place value tokens exchanging will need to be addressed. 	Use part wholes and bar models to demonstrate understanding of division steps. Times table and corresponding division facts will be essential to complete these methods. 	Use short division to solve problems $52 \div 4 = 13$ **Short division** 98 ÷ 7 becomes  Answer: 14
Division 2 digit by 1 digit with remainders	Use counters, cubes, objects to share into equal groups. When using base 10 and place value tokens exchanging will need to be addressed.	Use part wholes and bar models to demonstrate understanding of division steps.	Use short division to solve the problems. $53 \div 4 = 13\ r1$

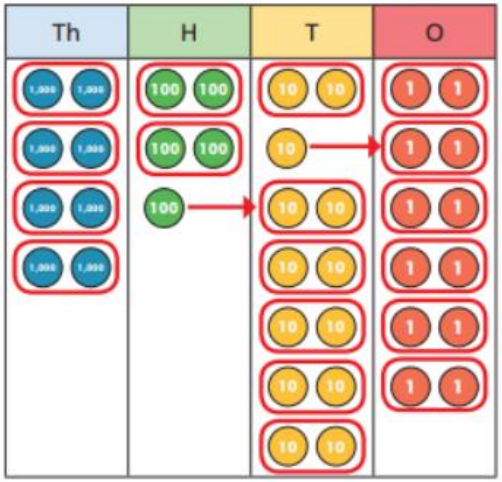
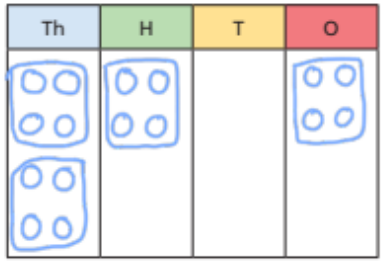
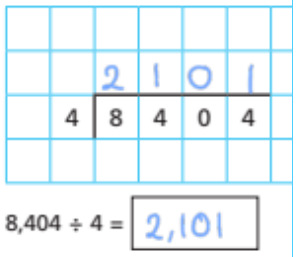
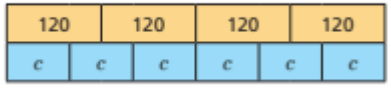
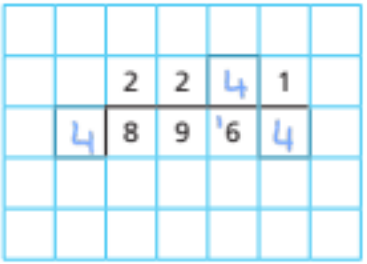
	 	 	432 ÷ 5 becomes 8 6 r 2 3 5 4 3 2 Answer: 86 remainder 2
Dividing 3 digits by 1 digit	Place value counters and base 10 can be used to share into equal groups. 	Part whole models and bar models can be used to demonstrate division. 	Short division should be used to solve these problems. 36 ÷ 4 = 9 37 ÷ 4 = 9r1 38 ÷ 4 = 9r2 39 ÷ 4 = 9r3 40 ÷ 4 = 10

			$\begin{array}{r} 32 \text{ r } 1 \\ 3\overline{)97} \end{array}$

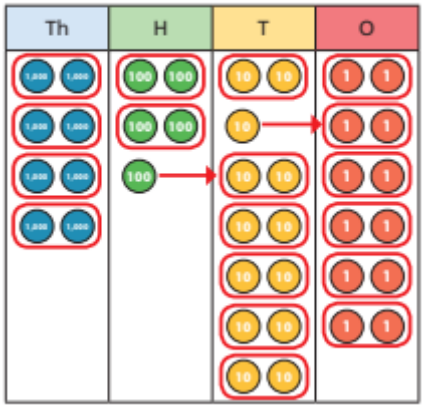
Year 5 Division			
Objective	**Concrete**	**Pictorial**	**Abstract**
Division 2 by one digit No remainders **RECAP OF YR4 METHOD**	Use counters, cubes, objects to share into equal groups. When using base 10 and place value tokens exchanging will need to be addressed.  	Use part wholes and bar models to demonstrate understanding of division steps. Times table and corresponding division facts will be essential to complete these methods.   	Use short division to solve problems $52 \div 4 = 13$ **Short division** 98 ÷ 7 becomes  Answer: 14
Division 2 digit by 1 digit with remainders	Use counters, cubes, objects to share into equal groups. When using base 10 and place value tokens exchanging will need to be addressed.	Use part wholes and bar models to demonstrate understanding of division steps.	Use short division to solve the problems. $53 \div 4 = 13\ r1$

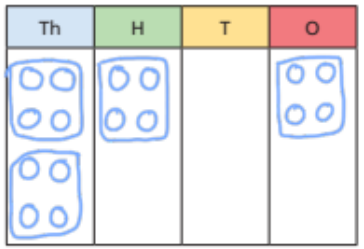
RECAP ON YR4 METHOD	 	 	432 ÷ 5 becomes 8 6 r 2 5 4 3 2 3 Answer: 86 remainder 2
Dividing 3 digits by 1 digit **RECAP OF YR4 METHOD**	Place value counters and base 10 can be used to share into equal groups. 	Part whole models and bar models can be used to demonstrate division. 	Short division should be used to solve these problems. 36 ÷ 4 = 9 37 ÷ 4 = 9r1 38 ÷ 4 = 9r2 39 ÷ 4 = 9r3 40 ÷ 4 = 10

			
Divide 4 digit numbers by 1 digit number	Concrete methods can be used alongside abstract methods to ensure understanding and fluency. 	Allow children to draw the problems to gain mastery and fluency within division.    	Use short division method to solve problems and link with pictorial and abstract to provide context and understanding.  

Year 6 Division			
Objective	**Concrete**	**Pictorial**	**Abstract**
Divide 4 digit numbers by 1 digit number **RECAP OVER YR5**	Concrete methods can be used alongside abstract methods to ensure understanding and fluency.  	Allow children to draw the problems to gain mastery and fluency within division.   8,404 ÷ 4 = 2,101 Books are available to buy in three different deals.  Which is the best deal? Deal B  c = 80	Use short division method to solve problems and link with pictorial and abstract to provide context and understanding.  Find the missing digits. a) 
Divide numbers of up to 4 digits by 2 digits	Use place value counters alongside the abstract to help secure understanding and give context to the problems.	Use place value grids to help support understanding alongside the abstract- children can draw the problem.	Use short division to solve problems

a) 8,532 ÷ 2 = 4,266





		2	1	0	1
	4	8	4	0	4

8,404 ÷ 4 = 2,101

496 ÷ 11 = 45 r1

$$11 \overline{)\,4\,9\,^{5}6} \qquad 45\text{ r }1$$

	0	4	8	9
15	7	$^{7}3$	$^{13}3$	$^{13}5$

Long Division

Use long division to solve these problems.

432 ÷ 12 = 36

		0	3	6	
1	2	4	3	2	
	–	3	6	0	(×30)
			7	2	
	–		7	2	(×6)
				0	

$12 \times 1 = 12$
$12 \times 2 = 24$
$12 \times 3 = 36$
$12 \times 4 = 48$
$12 \times 5 = 60$
$12 \times 6 = 72$
$12 \times 7 = 84$
$12 \times 8 = 96$
$12 \times 7 = 108$
$12 \times 10 = 120$

7,335 ÷ 15 = 489

	0	4	8	9	
15	7	3	3	5	
–	6	0	0	0	(×400
	1	3	3	5	
–	1	2	0	0	(×80)
		1	3	5	
–		1	3	5	(×9)
				0	

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

372 ÷ 15 = 24 r12

			2	4	r	1	2
1	5	3	7	2			
	–	3	0	0			
			7	2			
	–		6	0			
			1	2			

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

$$372 \div 15 = 24\frac{4}{5}$$

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	–	3	0	0	
			7	2	
	–		6	0	
			1	2	

432 ÷ 15 = 28·8

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      28·8
   ┌──────
15 ) 432·0
     30↓
     132
     120↓
      120
      120
        0
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Glossary

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor